

**A WAVE ANALYZER EMPLOYING VARIABLE  
SPEED MAGNETIC TAPE**

---

**James Vernon Haley  
and  
Jeremiah E. Lenihan**













A WAVE ANALYZER EMPLOYING VARIABLE SPEED MAGNETIC TAPE

by

JAMES V. HALEY, Lieutenant (junior grade), U. S. Navy  
B.S., U.S. Naval Academy (1949)

JEREMIAH E. LENIHAN, Lieutenant, U.S. Navy  
B.S., U.S. Naval Academy (1946)

8854

on s. line:

HALEY

1954

TH 1110

H14

FILLMENT

R THE

DEER

IE OF

Letter on front cover:

A WAVE ANALYZER EMPLOYING VARIABLE

ABLE SPEED MAGNETIC TAPE

James Vernon Haley  
and  
Jeremiah E. Lenihan



A W

JAME

8854  
on s, ine:

HALEY

1954

TH 1910  
H14

Letter on front cover:

A WAVE ANALYZER EMPLOYING V VI-

ABLE SPEED MAGNETIC TAPE

James Vernon Haley  
and  
Jeremiah E. Lenihan

Accoy



A WAVE ANALYZER EMPLOYING VARIABLE SPEED MAGNETIC TAPE

by

JAMES V. HALEY, Lieutenant (junior grade), U. S. Navy  
B.S., U.S. Naval Academy (1949)

JEREMIAH E. LENIHAN, Lieutenant, U.S. Navy  
B.S., U.S. Naval Academy (1946)

SUBMITTED IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE  
DEGREE OF NAVAL ENGINEER

at the

MASSACHUSETTS INSTITUTE OF  
TECHNOLOGY

June, 1954

---





Cambridge, Massachusetts  
May 24, 1954

Professor L. F. Hamilton  
Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Massachusetts

Dear Professor Hamilton:

In accordance with the requirements for the degree  
of Naval Engineer, we submit herewith a thesis entitled  
"A Wave Analyzer Employing Variable Speed Magnetic Tape".

Respectfully,

---

James V. Haley  
Lieutenant (junior grade),  
U. S. Navy

---

Jeremiah E. Lenihan  
Lieutenant,  
U. S. Navy

85064

25232

THE UNIVERSITY OF CHICAGO  
CHICAGO, ILL.

THE UNIVERSITY OF CHICAGO  
CHICAGO, ILL.  
CHICAGO, ILL.

CHICAGO, ILL.

CHICAGO, ILL.

CHICAGO, ILL.

CHICAGO, ILL.

CHICAGO, ILL.

# A WAVE ANALYZER EMPLOYING VARIABLE SPEED MAGNETIC TAPE

by

James V. Haley, Lieutenant (junior grade) U. S. Navy  
Jeremiah E. Lenihan, Lieutenant, U. S. Navy

Submitted to the Department of  
Naval Architecture and Marine Engineering  
May 24, 1954

in partial fulfillment of the requirements for the  
degree of Naval Engineer

## ABSTRACT

The object of this thesis is to make a preliminary investigation of a new type wave analyzer. The proposed wave analyzer would employ a variable speed magnetic tape to effect a multiplication in frequency. The resultant analysis would be made with a constant-percentage resolution.

A basic analyzer theory has been formulated. Specific theoretical results include: (1) determination of two multiplier speed-time relationships, a minimum-analysis-time solution, and an equal-analysis-sample solution, (2) determination of the required length of sample tape which is found to be a function of the desired percentage resolution, and (3) evaluation of the response of a simple selective network to a frequency excitation which varies very nearly linearly with time while within the pass-band. Dynamic amplitude and frequency distortion will occur as a result of the variable frequency excitation. It is shown that these analyzer errors can be predicted by use of a single parameter.

An experimental investigation of the analyzer principles was attempted using magnetic tape wound around a disc. The speed of the disc was controlled by a Ward-Leonard system. Other experiments were made with the speed of the disc slowing down due to its own damping. Experimental results were limited by: (1) variation in the distance between the reproduce head and the magnetic tape due to the eccentricity of the disc, and (2) inability to control accurately the speed of the disc.

Within the pass-band the deviation from a linear sweep for both multiplier functions is less than  $1/Q$ . It is concluded that linear-frequency-sweep theory is applicable to the proposed analyzer. Furthermore, it is concluded that equal sample analysis is a desirable feature in order to avoid a possible time-distribution ambiguity in measurements. However, the required multiplication function unfavorably affects total analysis time requirements and analyzer complexity.

Thesis Supervisor: Thomas F. Jones, Jr.  
Title: Assistant Professor of  
Electrical Engineering

THE UNIVERSITY OF CHICAGO  
 DIVISION OF PHYSICAL SCIENCES  
 DEPARTMENT OF PHYSICS  
 5712 S. DICKINSON AVE.  
 CHICAGO, ILL. 60637

**ABSTRACT**

The subject of this thesis is an investigation of a new type of speed measurement. The method used was a variable speed magnetic tape to effect a modification in frequency. The resulting analysis was made with a constant-frequency resolution.

A single amplifier theory has been developed. The results of this investigation are (1) a determination of the relationship between the speed of the tape and the frequency of the signal, and (2) a determination of the relationship between the speed of the tape and the frequency of the signal. The results of this investigation are (1) a determination of the relationship between the speed of the tape and the frequency of the signal, and (2) a determination of the relationship between the speed of the tape and the frequency of the signal.

In the experimental investigation of the speed of the tape, the speed of the tape was varied by a factor of two. The results of this investigation are (1) a determination of the relationship between the speed of the tape and the frequency of the signal, and (2) a determination of the relationship between the speed of the tape and the frequency of the signal.

With the use of the method described in this thesis, the speed of the tape was varied by a factor of two. The results of this investigation are (1) a determination of the relationship between the speed of the tape and the frequency of the signal, and (2) a determination of the relationship between the speed of the tape and the frequency of the signal.

This is a report of the work done by the author during the period from 1964 to 1966. The work was done at the University of Chicago, Division of Physical Sciences, Department of Physics, 5712 S. Dickinson Ave., Chicago, Ill. 60637.

#### ACKNOWLEDGEMENTS

The authors thank Professor T. F. Jones, Jr., ever burdened by a heavy schedule in his own department, for his supervision and counsel. The basic concept of this thesis originated with Dr. J. W. Horton, Chief Research Consultant, U. S. Navy Underwater Sound Laboratory, New London, Connecticut. Dr. Horton also was helpful in arranging for equipment. The authors are obligated to the U.S. Navy Underwater Sound Laboratory for equipment necessary for the progress of the thesis. The Acoustics Laboratory at M.I.T. assisted in providing additional equipment and suggestions. The skill of our typist, Miss Frances Doherty, is evident in the following pages.

Traces of the original document are evident in the following pages.

## TABLE OF CONTENTS

I	INTRODUCTION	1
	The Basic Analyzer	2
	Wave Analyzers in General	5
	Wave Analyzers Employing Magnetic Media	6
	Variable Frequency Excitation of Selective Networks	9
	Scope of the Thesis	11
II-A	DEVELOPMENT OF ANALYZER THEORY	12
	The Multiplier Speed-Time Relationship	15
	The Response of the Fixed Selective Network to a Varying Frequency Excitation	17
	Magnetic Recording Characteristics	20
	Sample Length Considerations	22
	Summary	22
II-B	EXPERIMENTAL PROCEDURE	23
	Outline of Operation of Block Diagram	23
III	RESULTS	28
	Theoretical Results	28
	Experimental Results	28
IV	DISCUSSION AND CONCLUSIONS	40
	Discussion of Theoretical Results	40
	Discussion of Experimental Results	42
	Conclusions	45
	Recommendations	46
Appendix A	NOMENCLATURE	49
Appendix B	THEORETICAL DETERMINATION OF MULTIPLIER SPEED- TIME RELATIONSHIP	51
	The Minimum-Analysis-Time Solution	51
	A Linear Multiplier	55
	The Equal-Sample-Analysis Solution	58
	The Excitation Function	61
	Theoretical Evaluation of the System Response	64

11

62

22

15

2

6

192

2000

11/2/20

100



Appendix C	AMBIGUITY IN WAVE ANALYLER MEASUREMENTS	72
Appendix D	SAMPLE LENGTH CONSIDERATIONS	77
	An Alternative Approach	78
	Butt-Weld Considerations	79
Appendix E	DETAILS OF EXPERIMENTAL EQUIPMENT AND PROCEDURE	81
	Introduction	
	Disc and Reproduce Head Assembly	
	Ward-Leonard System	
	Selective System and Detector	
	Amplifier	
	Frequency Measurements	
	Possible Equipment	
Appendix F	BIBLIOGRAPHY	103



# FIGURES AND TABLES

Figure	1.1	Basic Analyzer Block Diagram	4
	2.1	Basic Physical Considerations of Multiplier Action	14
	2.2	" " " " " "	14
	2.3	" " " " " "	14
	2.4	Transmission of a Simple Selective Network in Continuous Frequency Analysis	19
	2.5	Block Diagram of Driven System	26
	2.6	Block Diagram of Free-Running System	27
	3.1	Multiplier Speed-Time Relationships, I	31
	3.2	Multiplier Speed-Time Relationships, II	32
	3.3	Peak Transmission Power Error for Continuous Analysis of Simple Selective Network	33
	3.4	Frequency Lag in Peak Transmission for Continuous Analysis	34
	3.5	Relationship Between Effective Band-Width and Nominal Band-Width	35
	3.6	Analyzer Indication of Recorded Components	36
	3.7	Expanded Indication of Two Recorded Components Showing Resolution of Analyzer	37
	3.8	Free Running Speed-Time Relationship	38
	3.9	Output Pulse Shapes for Different Q's	39
	B.1	A Linear Speed Time Relationship	56
	B.2	Linear Frequency Multiplication Output	56
Table	B.1	Characteristics of 3 Multiplier Relationships	62
	B.2	$\theta$ Expressed as a Maclaurin's series	63
	B.3	Trigonometric Identities	63
Figure	B.3	Selective System Used for Response Evaluation	66
	c.1	Response of Band-Pass Filter to Propeller Sounds	73
	E.1	Disc Assembly and Drive Motor Assembly	82
	E.2	Reproduce Head Mounting	83
	E.3	Effect of Disc Eccentricity on Reproduced Frequency	86
	E.4	Butt Joint Effect	86
	E.5	Effect of Tape Surface Irregularities	87
	E.6	Effect of Air Gap Variation on Reproduced Frequency	87
	E.7	Servo Amplifier - Voltage Section	90
	E.8	Servo Amplifier - Power Section	91
	E.9	Drum Speed as Function of Controlling Voltage - Open Loop	92
	E.10	Variation of Filter Output with Constant Controlling Voltage	94
	E.11	Variation of Filter Output with all Parameters Remain- ing Constant	95
	E.12	Q Multiplier, Detector, and Marker Detector.	97

1	1.1	Introduction
2	1.2	Objectives of the Study
3	1.3	Scope of the Study
4	1.4	Organization of the Report
5	2	Background Information
6	2.1	General Background
7	2.2	Specific Background
8	2.3	Relevant Literature
9	2.4	Methodology
10	2.5	Experimental Setup
11	2.6	Data Collection
12	2.7	Results and Discussion
13	2.8	Conclusions
14	2.9	References
15	2.10	Appendices
16	2.11	Bibliography
17	2.12	Index
18	2.13	Glossary
19	2.14	Acronyms
20	2.15	Figures
21	2.16	Tables
22	2.17	Equations
23	2.18	Units
24	2.19	Notation
25	2.20	Summary
26	2.21	Concluding Remarks
27	2.22	Acknowledgments
28	2.23	References
29	2.24	Appendices
30	2.25	Bibliography
31	2.26	Index
32	2.27	Glossary
33	2.28	Acronyms
34	2.29	Figures
35	2.30	Tables
36	2.31	Equations
37	2.32	Units
38	2.33	Notation
39	2.34	Summary
40	2.35	Concluding Remarks
41	2.36	Acknowledgments
42	2.37	References
43	2.38	Appendices
44	2.39	Bibliography
45	2.40	Index
46	2.41	Glossary
47	2.42	Acronyms
48	2.43	Figures
49	2.44	Tables
50	2.45	Equations
51	2.46	Units
52	2.47	Notation
53	2.48	Summary
54	2.49	Concluding Remarks
55	2.50	Acknowledgments
56	2.51	References
57	2.52	Appendices
58	2.53	Bibliography
59	2.54	Index
60	2.55	Glossary
61	2.56	Acronyms
62	2.57	Figures
63	2.58	Tables
64	2.59	Equations
65	2.60	Units
66	2.61	Notation
67	2.62	Summary
68	2.63	Concluding Remarks
69	2.64	Acknowledgments
70	2.65	References
71	2.66	Appendices
72	2.67	Bibliography
73	2.68	Index
74	2.69	Glossary
75	2.70	Acronyms
76	2.71	Figures
77	2.72	Tables
78	2.73	Equations
79	2.74	Units
80	2.75	Notation
81	2.76	Summary
82	2.77	Concluding Remarks
83	2.78	Acknowledgments
84	2.79	References
85	2.80	Appendices
86	2.81	Bibliography
87	2.82	Index
88	2.83	Glossary
89	2.84	Acronyms
90	2.85	Figures
91	2.86	Tables
92	2.87	Equations
93	2.88	Units
94	2.89	Notation
95	2.90	Summary
96	2.91	Concluding Remarks
97	2.92	Acknowledgments
98	2.93	References
99	2.94	Appendices
100	2.95	Bibliography

## CHAPTER I

### INTRODUCTION

The objective of this thesis is the investigation of a new type of electronic wave analyzer.\* The proposed analyzer would employ a variable-speed magnetic medium to effect a multiplication in frequency. This multiplication in frequency would be employed in a manner analogous to the addition of frequencies in the conventional wave analyzer. As in such analyzers components of various frequencies are measured by sweeping a derived component across a fixed selective network. The advantage of frequency multiplication over frequency addition is that the resultant analysis may be made with a constant-percentage resolution\*\* rather than with a percentage resolution which varies between wide limits.

The device is intended to assist in the processing and analysis of data available in the form of magnetically recorded signals. The data will have been recorded at some constant speed. An automatic analysis will be effected by continuously varying the reproduce speed of the magnetic medium. Minimum analysis time is desired. Hence, the reproduce speed would be varied at the highest rate consistent with the limitations imposed by: (1) the response of the fixed selective network to an input of varying frequency, and (2) the specification of certain

---

\* This analyzer was proposed to the authors by Doctor J. W. Horton, Chief Research Consultant, U.S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Connecticut.

\*\* By resolution is meant the ability to distinguish between two closely adjacent frequency components. Percentage resolution is defined as resolution (measured in cycles per second) expressed as a percentage of mid-band frequency of the selective network.



desired analyzer characteristics which will be proposed in Chapter II-A. Automatic operation coupled with minimum analysis time would do much to alleviate the laborious and time-consuming analysis which is now necessary using commercially available wave analyzers.

Ideally, the analyzer should be able to consider and process signals lying within the frequency spectrum from 1 cycle to 100 kilocycles. Practically, certain characteristics of present-day recording techniques will limit the analysis spectrum-band. It is anticipated that at least two frequency-bands will be required to complete the analysis.

With the ever-increasing desire to investigate phenomena from a spectrum analysis consideration there is a decided need for equipment which would facilitate such measurements. In addition, there is a real need in the field of underwater sound for a constant percentage resolution analyzer which would incorporate automatic operation and rapidity of measurement. It is believed that similar needs exist in other fields.

In a sense the proposed analyzer will be composed of components which have been investigated at some length and which are in common use today. These components will be arranged, and will be caused to function in a manner to accomplish a specific desired result: spectrum analysis. This thesis will emphasize the inter-relationship between these components in order to accomplish this purpose, and will investigate the limitations which various components put on the problem and the system as a whole. It is not our intention to build a prototype wave analyzer. The thesis will attempt to answer as far as possible the question of the feasibility of the proposed analyzer.

### The Basic Analyzer

The proposed analyzer could be divided into four functional sub-

1. The first part of the report

2. The second part of the report

3. The third part of the report

4. The fourth part of the report

5. The fifth part of the report

6. The sixth part of the report

7. The seventh part of the report

8. The eighth part of the report

9. The ninth part of the report

10. The tenth part of the report

11. The eleventh part of the report

12. The twelfth part of the report

13. The thirteenth part of the report

14. The fourteenth part of the report

15. The fifteenth part of the report

16. The sixteenth part of the report

17. The seventeenth part of the report

18. The eighteenth part of the report

19. The nineteenth part of the report

20. The twentieth part of the report

21. The twenty-first part of the report

22. The twenty-second part of the report

23. The twenty-third part of the report

24. The twenty-fourth part of the report

25. The twenty-fifth part of the report

26. The twenty-sixth part of the report

27. The twenty-seventh part of the report

28. The twenty-eighth part of the report

29. The twenty-ninth part of the report

30. The thirtieth part of the report



divisions: (1) a frequency multiplier, (2) a fixed selective network, (3) an indicator, and (4) a control and programming section. A postulated arrangement is presented in Figure 1.1.

The signal to be analyzed is recorded at some constant speed on either magnetic tape or on the outer edge of a magnetic disc. This closed loop of predetermined length is constrained by the frequency multiplier to follow an appropriate speed-time relationship,  $N(t)$ .<sup>\*</sup> Such a multiplication might be achieved by: (1) continuously varying the speed of a magnetic disc, (2) wrapping magnetic tape around the periphery of a non-magnetic disc which proceeds at a variable speed, and (3) transporting a constant speed magnetic medium over a reproduce head array which rotates in accordance with  $N(t)$ .

Following the multiplier a reproduce system converts the recorded signals to an electrical excitation which sweeps across a fixed selective network. Selectivity is provided by a single, high - Q, band-pass filter per frequency-band. Next the signals proceed to the indicator, pass through appropriate averaging circuits, and finally are presented in the form of a permanent visual record. If simultaneous analysis of the entire spectrum is desired, one graphic-recorder channel per frequency-band is required.

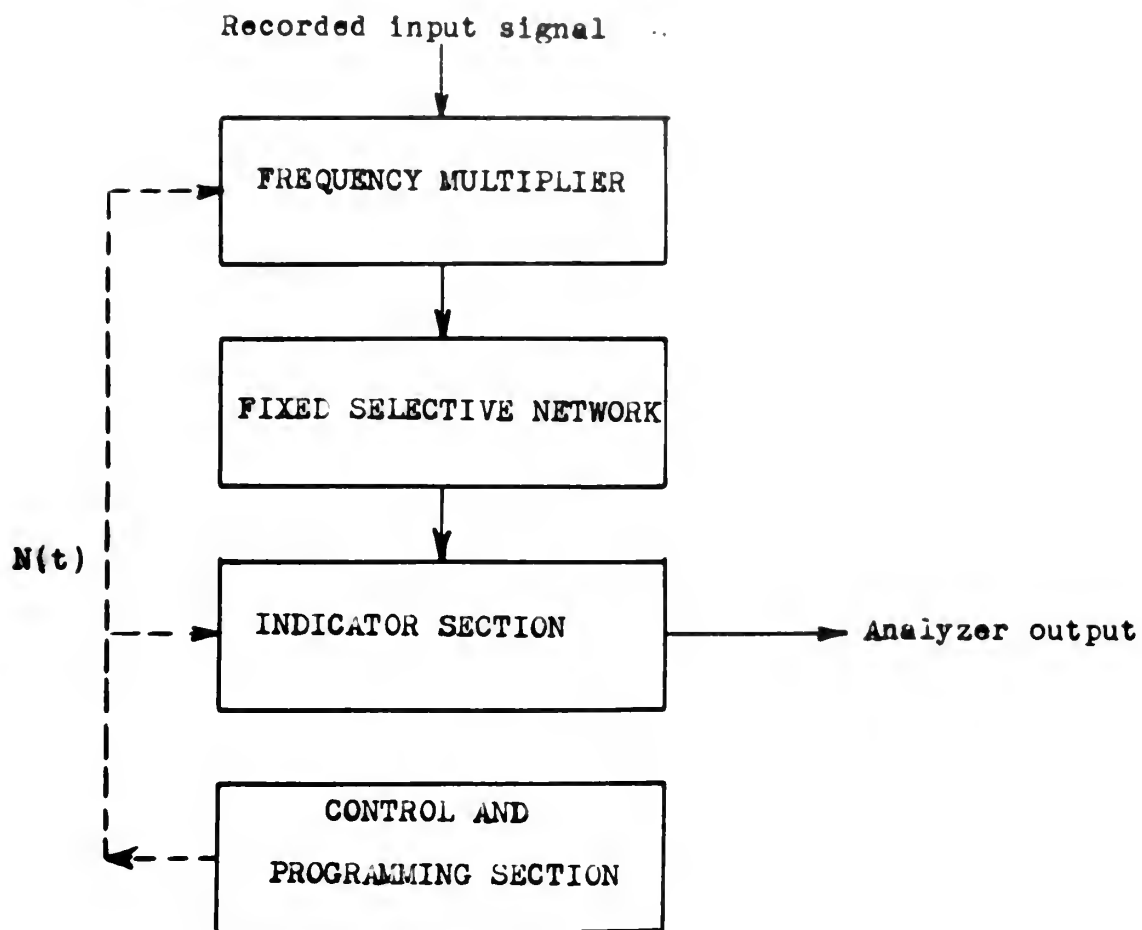
Note that the speed-time relationship,  $N(t)$ , is a fundamental link between three main analyzer sections. An appropriate tie-in must exist between the multiplier and the indicator. There is a possibility that the averaging circuits might also be a function of  $N(t)$ . It appears that the control and programming section will require use of servomechanisms combined with suitable electronic control circuits.

---

<sup>\*</sup> Appendix A provides a table of symbols and definitions.



FIGURE 1.1  
BASIC ANALYZER BLOCK DIAGRAM



———— path followed by analyzed signal

----- the multiplier speed time relationship,  $N(t)$



## Wave Analyzers In General

A thorough survey of available technical literature indicates that the proposed wave analyzer would have several unique characteristics. The basic novelty exists in the frequency multiplier. The variable speed magnetic medium effects a constant-percentage resolution analysis.

Beranek<sup>4</sup> presents a comprehensive study of the many different types of apparatus utilized to analyze a complex noise into a spectral distribution. Jastram and McCouch<sup>13</sup> have discussed the design requirements for a wave analyzer capable of measuring noise spectra in the video-frequency range. Their paper includes a discussion on the response of a resonant system to a non-periodic function. Another paper which concentrates on the design features of a spectrum analyzer was published recently by Soanes.<sup>19</sup> He discusses the increased number of problems associated with low frequency analysis where the time of one cycle becomes comparable with the total time available for the analysis.

Actually the plan to design a wave analyzer employing a frequency multiplying device with a fixed selective system is not a new one. In 1924 Sacia<sup>18</sup> proposed a non-automatic analyzer which was very accurate, but which required a great deal of analysis time. The sound waves were recorded on a strip of film, joined into an endless band, and then picked off with the aid of a photo-cell. Following the photo-cell was an amplifier containing a tuned electrical circuit. The speed of the film was varied so that the different partial tones of the sound wave were recorded singly, and an analysis was obtained. Similarly Barber and Ursall<sup>1,2</sup> have designed a wave analyzer used for examining spectra of ocean waves.

---

<sup>4</sup> Subscript numerals refer to similarly numbered entries in the Bibliography.



The record in the form of a variable-area black trace on a white background is wrapped around the periphery of a wheel. The wheel is made to revolve, and an electrical output is obtained from an optical device which views the record through a narrow slit close to the wheel. This output, a continual repetition of the recorded trace, is made to drive a sharply resonant vibration galvanometer. The successive resonances are recorded by a pen on moving paper. It is noted that its analysis time is about five minutes per octave; this is considered excessive for our wave analyzer.

Apparently the technique of designing a workable automatic wave analyzer has just recently been established.\* As recently as 1949 Barber<sup>2</sup> suggested a slow mechanical drive combined with continuous recording of analyzer energy output in order to expedite the spectrum analysis. His paper includes a detailed study on the optimum performance of such an analyzer.

#### Wave Analyzers Employing Magnetic Media

Magnetic recording processes are rapidly being applied in the instrumentation field. Uses include pulse systems, and carrier systems. Furthermore, conventional magnetic recorders employing high-frequency bias are being used for recording a band-width within the range of 100 cycles to 100 kilocycles.<sup>27</sup>

Certain research and industrial applications of magnetic tape have been previously restricted by limitations of the medium itself.<sup>26</sup> Recently tape manufacturers have made enormous progress in the quality of their products which are designed for research applications.<sup>23</sup> These advances have been stimulated largely by the exacting demands of the

---

\* By automatic we mean that the analyzer is self acting, and that a continuous, permanent record of its output is made available.

10-10-60

The first of these is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test. The second is the fact that the data are not independent. This is evident from a visual inspection of the data and from a statistical test. The third is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The fourth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test. The fifth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The sixth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The seventh is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The eighth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The ninth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The tenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The eleventh is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The twelfth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The thirteenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The fourteenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The fifteenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The sixteenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The seventeenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The eighteenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The nineteenth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.

The twentieth is the fact that the data are not normally distributed. This is evident from a visual inspection of the data and from a statistical test.



of the computer field for improved magnetic tape memory devices.

Why use magnetic recording and reproduction techniques in wave analyzers? The answer to this question is contained within two characteristic advantages of magnetic media. First, there are those general functions characteristic of magnetic recording: (1) recording and storage of signals for reproduction at will, and (2) improved signal-to-noise ratio.

An excessive length of analysis time is required using conventional methods. It would be desirable to accelerate the rate of scanning. This can be accomplished by use of magnetic media. Its second characteristic advantage is contained in the following discussion. A magnetic pattern is impressed in a medium traveling at a given velocity. If it is then passed over a pickup device at the same velocity, a characteristic wave length will be observed which is equal to the wave length originally recorded on the tape. Next, if the medium velocity is doubled over the recording velocity, the reproduce head sees two wave lengths in the same length of time it originally saw only one. As a result, the frequency obtained during play-back is twice that which was originally recorded. This characteristic can be expressed in a general form as

$$\frac{S}{S_r} = \frac{f_b}{f_a} = N$$

where

$S$  = the instantaneous reproduce speed.

$S_r$  = the original recording speed.

$f_a$  = the recorded frequency component.

$f_b$  = the frequency observed at the reproduce head  
when the magnetic medium is traveling at speed  $S$ .

$N$  = the instantaneous reproduce speed or reproduce  
frequency ratio

Page 1

Dear Sir,

I have the honor to acknowledge the receipt of your letter of the 15th inst.

in relation to the matter of the proposed extension of the term of office of the

Commissioners of the General Land Office, and in reply to inform you that the

same has been referred to the proper authorities for their consideration.

I am, Sir, very respectfully, your obedient servant,

Very truly yours,

John D. Smith, Secretary of the Interior.

Enclosed for you are two copies of the report of the

Commissioners of the General Land Office, dated the 10th inst.

in relation to the proposed extension of the term of office of the

Commissioners of the General Land Office, and in reply to inform you that the

same has been referred to the proper authorities for their consideration.

I am, Sir, very respectfully, your obedient servant,

John D. Smith,  
Secretary of the Interior.

Very truly yours,

John D. Smith, Secretary of the Interior.

Enclosed for you are two copies of the report of the

Commissioners of the General Land Office, dated the 10th inst.

in relation to the proposed extension of the term of office of the

Commissioners of the General Land Office, and in reply to inform you that the

same has been referred to the proper authorities for their consideration.

Very truly yours,

This multiplication feature allows the original frequency spectrum to be converted to an equivalent spectrum in a higher frequency range. A wider band-width analyzing filter results with its subsequent lower build-up time.<sup>9</sup> The effective band-width of the analyzer filter, when referred to the original frequency spectrum, is the actual filter band-width divided by the reproduce speed ratio,  $N$ .

For any fixed reproduce speed,  $S$ , the rate of scanning is increased, and total analysis time is decreased. This comes from the requirement that for linear sweep excitation the ratio of sweeping is directly proportional to the square of the bandwidth.<sup>19</sup>

There are several automatic wave analyzers which employ a magnetic medium to effect a multiplication of frequency spectra. Two such commercial analyzers are Bell Telephone Laboratories' Sound Spectrograph,<sup>23,26</sup> and Kay Electric Company's Vibralyzer.<sup>20</sup> These devices only make limited use of this important multiplication feature. In each case the medium reproduce speed is maintained at some constant value, which is usually only 3 to 5 times the record speed. Instead of making  $N$  a function of time, as is the case for the proposed analyzer, the analysis is obtained by use of a heterodyne process. No known analyzer achieves frequency selectivity by use of a continuously varying reproduce speed. In this respect the proposed analyzer is unique.

The Sound Spectrograph, also known commercially as the Sona-Graph, is a wave analyzer which produces a permanent visual record showing the distribution of energy in both frequency and time. Audio sounds up to 8000 cps can be studied. A typical recorded sample length is from 2 to 3 seconds duration. The signal to be analyzed is recorded on a loop of magnetic tape which is mounted on a rotating disc. Analysis

The first of these is the fact that the  
 second of these is the fact that the  
 third of these is the fact that the  
 fourth of these is the fact that the  
 fifth of these is the fact that the  
 sixth of these is the fact that the  
 seventh of these is the fact that the  
 eighth of these is the fact that the  
 ninth of these is the fact that the  
 tenth of these is the fact that the  
 eleventh of these is the fact that the  
 twelfth of these is the fact that the  
 thirteenth of these is the fact that the  
 fourteenth of these is the fact that the  
 fifteenth of these is the fact that the  
 sixteenth of these is the fact that the  
 seventeenth of these is the fact that the  
 eighteenth of these is the fact that the  
 nineteenth of these is the fact that the  
 twentieth of these is the fact that the  
 twenty-first of these is the fact that the  
 twenty-second of these is the fact that the  
 twenty-third of these is the fact that the  
 twenty-fourth of these is the fact that the  
 twenty-fifth of these is the fact that the  
 twenty-sixth of these is the fact that the  
 twenty-seventh of these is the fact that the  
 twenty-eighth of these is the fact that the  
 twenty-ninth of these is the fact that the  
 thirtieth of these is the fact that the  
 thirty-first of these is the fact that the  
 thirty-second of these is the fact that the  
 thirty-third of these is the fact that the  
 thirty-fourth of these is the fact that the  
 thirty-fifth of these is the fact that the  
 thirty-sixth of these is the fact that the  
 thirty-seventh of these is the fact that the  
 thirty-eighth of these is the fact that the  
 thirty-ninth of these is the fact that the  
 fortieth of these is the fact that the  
 forty-first of these is the fact that the  
 forty-second of these is the fact that the  
 forty-third of these is the fact that the  
 forty-fourth of these is the fact that the  
 forty-fifth of these is the fact that the  
 forty-sixth of these is the fact that the  
 forty-seventh of these is the fact that the  
 forty-eighth of these is the fact that the  
 forty-ninth of these is the fact that the  
 fiftieth of these is the fact that the  
 fifty-first of these is the fact that the  
 fifty-second of these is the fact that the  
 fifty-third of these is the fact that the  
 fifty-fourth of these is the fact that the  
 fifty-fifth of these is the fact that the  
 fifty-sixth of these is the fact that the  
 fifty-seventh of these is the fact that the  
 fifty-eighth of these is the fact that the  
 fifty-ninth of these is the fact that the  
 sixtieth of these is the fact that the  
 sixty-first of these is the fact that the  
 sixty-second of these is the fact that the  
 sixty-third of these is the fact that the  
 sixty-fourth of these is the fact that the  
 sixty-fifth of these is the fact that the  
 sixty-sixth of these is the fact that the  
 sixty-seventh of these is the fact that the  
 sixty-eighth of these is the fact that the  
 sixty-ninth of these is the fact that the  
 seventieth of these is the fact that the  
 seventy-first of these is the fact that the  
 seventy-second of these is the fact that the  
 seventy-third of these is the fact that the  
 seventy-fourth of these is the fact that the  
 seventy-fifth of these is the fact that the  
 seventy-sixth of these is the fact that the  
 seventy-seventh of these is the fact that the  
 seventy-eighth of these is the fact that the  
 seventy-ninth of these is the fact that the  
 eightieth of these is the fact that the  
 eighty-first of these is the fact that the  
 eighty-second of these is the fact that the  
 eighty-third of these is the fact that the  
 eighty-fourth of these is the fact that the  
 eighty-fifth of these is the fact that the  
 eighty-sixth of these is the fact that the  
 eighty-seventh of these is the fact that the  
 eighty-eighth of these is the fact that the  
 eighty-ninth of these is the fact that the  
 ninetieth of these is the fact that the  
 ninety-first of these is the fact that the  
 ninety-second of these is the fact that the  
 ninety-third of these is the fact that the  
 ninety-fourth of these is the fact that the  
 ninety-fifth of these is the fact that the  
 ninety-sixth of these is the fact that the  
 ninety-seventh of these is the fact that the  
 ninety-eighth of these is the fact that the  
 ninety-ninth of these is the fact that the  
 hundredth of these is the fact that the

results are marked on electrically sensitive paper. This record is mounted on a cylindrical drum, which is mechanically coupled to the tape loop.

The Vibralyzer furnishes a frequency analysis of low-frequency sounds in the range from 5 to 4400 cps. The signal to be analyzed is recorded on the outer edge of a magnetic disc. This closed-loop signal is scanned with each revolution of a recording drum. The output is graphically presented on a sheet of facsimile-type paper for a permanent visual record. Information is presented regarding time, frequency, and amplitude of the recorded signal.

#### Variable Frequency Excitation of Selective Networks

Preliminary investigation indicates that we will be concerned with the response of linear selective networks to driving functions in which the frequencies vary other than linearly with time. There is very little information available on such driving functions. The response of a LCR circuit to a logarithmic frequency-sweep driving function has been investigated by Panfield.<sup>17</sup> This theoretical development resulted from a lengthy, graphical evaluation of a convolution integral equation. Unfortunately, this study only provides information for a specific case out of the yet-unexplored family of response curves which exist for a logarithmic-frequency-sweep excitation.

On the other hand, there is a great deal of information available on the response of selective networks to linear-frequency-sweep excitation. A review of the technical literature will be provided below. Briefly these studies can be summarized as follows. The response of a selective network will be nearly identical to its steady-state response provided the sweep rate is very low. As the rate of sweeping is increased, circuit transients make the response appreciably different from that obtained



with slow sweep rates. There will exist a dynamic, amplitude-depression distortion. A frequency-displacement distortion will also be associated with the maximum response. At the same time the effective band-width is increased over its nominal value. This decreases the effective value of  $Q$ . Furthermore, a ringing phenomena following the resonant response peak can result. This can be characterized as the beat-tone between the damped oscillation of the selective circuit and the sweep frequency signal. The presence of these secondary maxima can completely mask the detection of weak components in the vicinity of a strong component.

Many authors have studied the response of resonant systems to excitation of a frequency varying linearly with time. Clavier<sup>5</sup> sought the mathematical condition for which the dynamic response is nearly the same as the static response. His solution indicates the need for very slow exploration speeds. Lewis<sup>14</sup> investigated the case of a mechanical shaft of linearly varying speed of rotation. His graphical solution involves fresnel integrals of complex argument which are not known to be tabulated to any extent even today. Hok<sup>10</sup> studied the response of a narrow-band resonant system. Hamilton's<sup>9</sup> solution to the problem involves the fourier analysis of the input and output spectra. Barber and Ursell<sup>2</sup> have presented a solution for the response of an oscillatory system to a tone whose frequency slowly increases or decreases. It is shown that the form of the response is very complicated, but that the variation of amplitude near resonance depends upon a single parameter involving the constants of the apparatus. Marique<sup>15</sup> investigated a sawtooth varying frequency excitation. He shows that the response is composed of two terms: one arising during each sweep, the other resulting from the preceding sweep. Soanes<sup>19,20</sup> completes the general problem by discussing the effects of starting the sweep in or near the pass-band. In addition,





he provides an excellent review of many different authors' approaches to the general problem of linear sweep excitation. In 1935, Meyer<sup>16</sup> provided an earlier correlation of similar studies. Ekstein and Schiffman<sup>6</sup> recently investigated the response of a linear network to an input with linearly variable frequency as obtained in sweep frequency testing.

### Scope of the Thesis

Time and economic considerations necessitate rather narrow limitations in the scope of the thesis. Obviously we cannot hope to design and construct a complete wave analyzer. Furthermore, there is considerable material available in the literature on magnetic reproduce systems, selective networks, averaging circuits, indicators, servomechanisms, and control circuits. This information will be readily available to the future designer. Hence, the thesis will concentrate on the unique features of the analyzer: the frequency multiplier and its associated speed-time relationship. These features raise certain questions which are not specifically answered in the literature:

- (1) What multiplier speed-time relationships are available which result in tolerable limits of distortion, minimum analysis time, and desired analyzer characteristics?
- (2) What is the proper relationship between desired percentage resolution and minimum analysis time per frequency band?
- (3) What limitations do magnetic recording characteristics impose upon the overall analyzer?
- (4) How many analysis frequency-bands should be used?
- (5) What length of sample is necessary for proper analysis?
- (6) What is the influence of  $N(t)$  upon the averaging, indicating, and programming sections?

1. The first of these is the fact that the  
2. second is the fact that the  
3. third is the fact that the  
4. fourth is the fact that the  
5. fifth is the fact that the  
6. sixth is the fact that the  
7. seventh is the fact that the  
8. eighth is the fact that the  
9. ninth is the fact that the  
10. tenth is the fact that the

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

## CHAPTER II-A

DEVELOPMENT OF ANALYZER THEORY

This chapter introduces a basic theory applicable to the proposed analyzer. The analysis is guided by specific questions raised in Chapter I.

Two fundamental requirements for any automatic wave analyzer are:

(1) to provide the greatest possible resolution of frequency components, and (2) to complete the analysis in the minimum possible time. Note that these are two conflicting aims. The smaller the pass-band of the selective network, the greater the resolution.<sup>3,19</sup> However, accurate output measurements are obtainable only if each component remains within this smaller pass-band for a longer period of time. Furthermore, it is possible (as is the case for this particular analyzer) that certain additional requirements, other than the characteristics of the selective network, will influence the determination of  $N(t)$ .

The Basic Selective System

The basic frequency selective system consists of a frequency multiplier, and a fixed selective network. Assume that the system input can be represented by a series of  $n$  discrete sinusoidal components, each of frequency  $f_{an}$ .<sup>\*</sup> The output of the multiplier is then a series of frequency modulated components,  $f_{bn}(t)$ . The multiplier function can be expressed in terms of the instantaneous magnetic-tape reproduce speed,  $S(t)$ , and the fixed recording speed,  $S_r$ .

$$N(t) = \frac{S(t)}{S_r} \quad (1)$$

---

\*Where subscript  $n$  takes on values 0, 1, 2, ...

[illegible]

*[Faint handwritten notes at the bottom of the page]*

The first of these is the fact that the  
 second of these is the fact that the  
 third of these is the fact that the  
 fourth of these is the fact that the  
 fifth of these is the fact that the  
 sixth of these is the fact that the  
 seventh of these is the fact that the  
 eighth of these is the fact that the  
 ninth of these is the fact that the  
 tenth of these is the fact that the

1. The first group of people who are interested in the study of the history of the United States are the people who are interested in the history of the United States.

2

A fundamental multiplier relationship exists:

$$f_{bn}(t) = f_{an}N(t) = f_{an} \frac{S(t)}{S_r} \quad (2)$$

It is appropriate to examine the physical operations involved in this system. A series of discrete components,  $f_{an}$ , have been recorded on magnetic tape at a constant recording speed,  $S_r$ . These are shown in Figure 2.1. It is logical to assume that the multiplier speed-time relationship will be in the form of a continuous, decaying function. Figure 2.2 is a sketch of such an assumed  $N(t)$ . Each recorded component,  $f_{an}$ , is multiplied by  $N(t)$ . Hence, the output of the multiplier is a family of decaying components,  $f_{bn}(t)$ , similar to those represented in Figure 2.3. These components sweep across the narrow-band selective network which ideally rejects all components lying outside the shaded area of Figure 2.3

The phenomena of frequency sweep must be considered from two different points of view. First, for any fixed input frequency (e.g.  $f_{a1}$ ), its derived component sweeps across the pass-band in a decaying fashion. For example, when

$$t = t_1, \quad f_{b1} = f_u$$

and, when

$$t = t_2, \quad f_{b1} = f_L$$

where  $f_u$  and  $f_L$  are the upper and lower cut-off frequencies for the selective network. On the other hand, derived components of lower-frequency recorded signals (e.g.  $f_{a1}$ ) are swept across the selective network before those derived components which are associated with higher-frequency recorded signals. (e.g.  $f_{a2}$ ).

The response of selective networks to linear-frequency-sweep excitation has been thoroughly investigated. (See discussion in Chapter I). Examination of our basic system fails to indicate that  $N(t)$  will necessarily

... ..

*Journal of Management Studies*, 19(6), 709-728.

16

... .. (9)

*Journal of Management Studies*, 36(7), 809-826.

THE UNIVERSITY OF CHICAGO LIBRARY

...the ... ..

... (faint text) ...

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

[illegible]

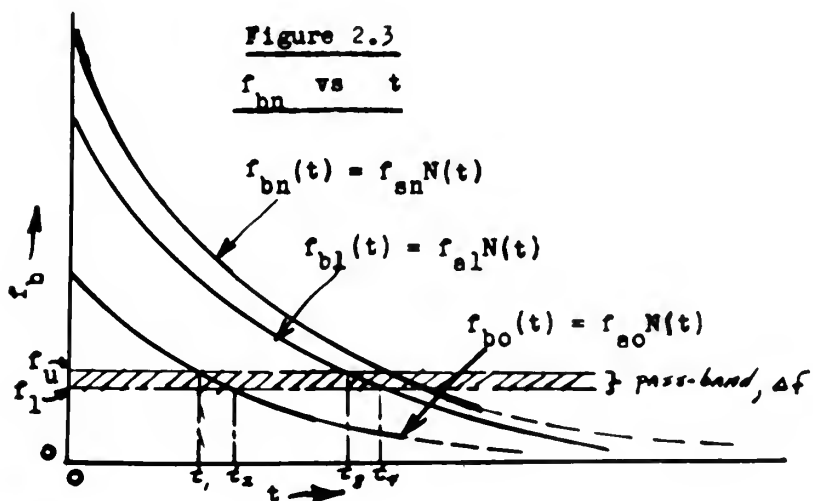
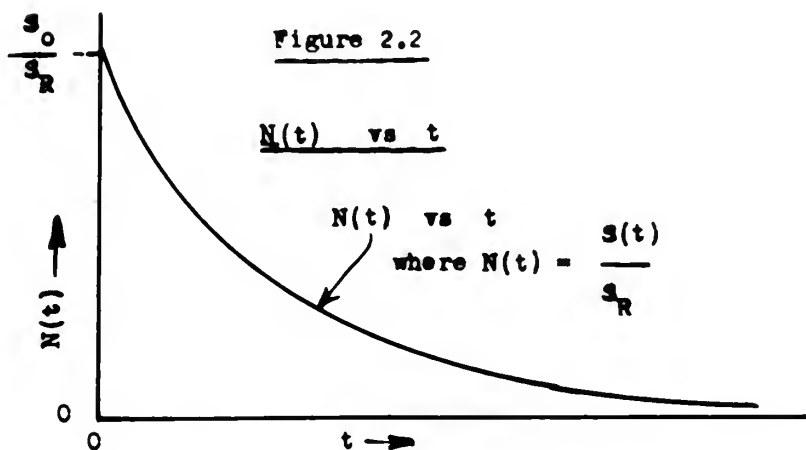
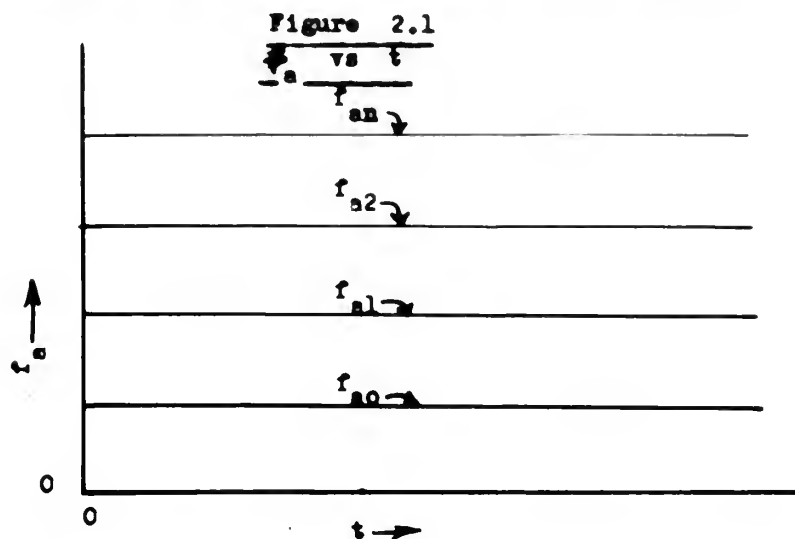
1

1. *Chlorophyll a* (Chl *a*)

1998

Figures 2.1 - 2.3

BASIC PHYSICAL CONSIDERATIONS OF MULTIPLIER ACTION







be a linear relationship. Nevertheless, this wave analyzer typically contains a narrow pass-band in order to obtain maximum frequency resolution. As a result,  $N(t)$  might properly be approximated as a linear sweep within this narrow frequency-band. Available linear-frequency-sweep research could then be applied as an aid in properly evaluating  $N(t)$ .

### The Multiplier Speed-Time Relationship

In Appendix B three theoretical multiplier speed-time relationships are derived by a qualitative approach.

First, the minimum-analysis-time solution is determined. It can be expressed in the form

$$S(t) = S_0 e^{-ct} \quad (11b)$$

where  $S_0$  = the value of  $S$  at  $t$  equals 0.

$$c = \frac{(\Delta f)^2}{K f_u}$$

$\Delta f$  = the width of the resonant response curve at the cut-off frequencies

$f_u$  = the upper cut-off frequency of the selective network.

$K$  = a pure numeric (cycles)

For this solution each derived component,  $f_{bm}$ , remains within the pass-band the minimum time for acceptable analysis. This condition is based on the requirement that the output of the selective system remains within certain tolerable limits of amplitude and frequency distortion.

Secondly, the equal-sample-analysis solution is determined. It can be expressed in the form

$$S(t) = \frac{S_0}{1 + ct} \quad (21b)$$

Each analyzed component remains under scrutiny for the entire length of sample. The energy reported at each frequency is associated with the same

1. (a)  $\frac{1}{2}$   
(b)  $\frac{1}{3}$

(c)  $\frac{1}{4}$

(d)  $\frac{1}{5}$

2. (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d)  $\frac{1}{5}$

3. (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d)  $\frac{1}{5}$

(e)  $\frac{1}{6}$

(f)  $\frac{1}{7}$

(g)  $\frac{1}{8}$

(h)  $\frac{1}{9}$

(i)  $\frac{1}{10}$

(j)  $\frac{1}{11}$

(k)  $\frac{1}{12}$

(l)  $\frac{1}{13}$

(m)  $\frac{1}{14}$

(n)  $\frac{1}{15}$

(o)  $\frac{1}{16}$

(p)  $\frac{1}{17}$

analysis sample as the energies reported for all other frequencies. This practical feature is very advantageous since a possible ambiguity in measurements is avoided. Appendix C develops these undesirable measurement errors which occur in conventional wave analyzers.

Thirdly, a linear speed-time relationship is developed, and can be expressed as

$$S(t) = S_0 \left[ 1 - \frac{c f_0 t}{(f_{an})_{max}} \right] \quad (15a)$$

Equation 15a has limited theoretical use. The solution only serves to correlate the previous non-linear relationships and the conventional linear-frequency-sweep excitation.

A fundamental and useful characteristic exists for both the minimum-analysis-time solution and the equal-sample-analysis solution. The maximum deviation from linearity within the pass-band of the selective network is less than  $\frac{1}{Q}^*$  where

$$Q = \frac{f_f}{\Delta f}$$

$f_f$  = the mid-band frequency of the selective network.

For all practical purposes the sweep rate is linear since the proposed analyzer will contain a high-Q selective network.

Figure 2.3 describes the family of decaying components,  $f_{bn}(t)$ , sweeping across the fixed pass-band. The minimum-analysis-time solution exhibits equal sweep rates at  $f_u$  for all values of  $f_{bn}(t)$ . Hence, if one rate is critical, all rates are equally critical. On the other hand, the equal-sample-analysis solution can have only one critical sweep rate at  $f_u$ ; this occurs for the derived component  $f_{bo}(t)$  associated with the lowest recorded frequency component,  $f_{ao}$ . All other sweep rates within the pass-band will oversatisfy the characteristic time requirements of the selective network.

\*

See Equations 14 and 23 in Appendix B.

... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...

(15a)

$$\left[ \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right]$$

... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...

$$\frac{1}{2} = \frac{1}{2}$$

... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...  
... the ... of the ...

## The Response of the Fixed Selective Network to a Varying Frequency Excitation

Two basic multiplier relationships are presented in Equations 11b and 21b. Both expressions are non-linear functions. Yet, within the pass-band of the selective network the excitation function

$$e(t) = E \cos \theta = E \cos \left( \frac{d\theta}{dt} t \right) = E \cos [2\pi f_{bm}(t)]$$

follows very nearly a linear sweep rate.

The three multiplier relationships can be expanded in a Maclaurin's Series in angular displacement,  $\theta$ , of the form

$$\theta = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \dots$$

The first three terms for two basic multiplier relationships are identically equal to the linear speed-time expansion resulting from Equation 15a. Within the pass-band the  $t^3$  and all subsequent terms are considered negligible.

In Appendix B the excitation of a simple LCR circuit is investigated. The response to a nearly linear sweep excitation function is evaluated by the real convolution integral

$$r(t) = \int_{-\infty}^t e(\tau) h(t - \tau) d\tau \quad (25)$$

where

$r(t)$  = the response of the system.

$e(t)$  = the excitation function of the system.

$h(t)$  = the unit impulse response of the linear system.

This integral is broken down analytically using the method of Barber and Ursell.<sup>2</sup> The derivation is included in the thesis since: (1) the solution provides a better understanding of the basic parameters involved, (2) the derivation conveys a physical picture of the response and (3) this approach provides a logical starting point for future studies concerned with the effects of ignoring the  $t^3$  and any subsequent terms in the expansion of  $\theta$ . Barber and Ursell studied a mechanical analogy to our

2. 1941 1942 1943 1944 1945 1946 1947 1948 1949 1950 1951 1952 1953 1954 1955 1956 1957 1958 1959 1960 1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025 2026 2027 2028 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 2075 2076 2077 2078 2079 2080 2081 2082 2083 2084 2085 2086 2087 2088 2089 2090 2091 2092 2093 2094 2095 2096 2097 2098 2099 2100 2101 2102 2103 2104 2105 2106 2107 2108 2109 2110 2111 2112 2113 2114 2115 2116 2117 2118 2119 2120 2121 2122 2123 2124 2125 2126 2127 2128 2129 2130 2131 2132 2133 2134 2135 2136 2137 2138 2139 2140 2141 2142 2143 2144 2145 2146 2147 2148 2149 2150 2151 2152 2153 2154 2155 2156 2157 2158 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2180 2181 2182 2183 2184 2185 2186 2187 2188 2189 2190 2191 2192 2193 2194 2195 2196 2197 2198 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2209 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2220 2221 2222 2223 2224 2225 2226 2227 2228 2229 2230 2231 2232 2233 2234 2235 2236 2237 2238 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2260 2261 2262 2263 2264 2265 2266 2267 2268 2269 2270 2271 2272 2273 2274 2275 2276 2277 2278 2279 2280 2281 2282 2283 2284 2285 2286 2287 2288 2289 2290 2291 2292 2293 2294 2295 2296 2297 2298 2299 2300 2301 2302 2303 2304 2305 2306 2307 2308 2309 2310 2311 2312 2313 2314 2315 2316 2317 2318 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2340 2341 2342 2343 2344 2345 2346 2347 2348 2349

Figure 1. The effect of the concentration of the  $\text{Ca}^{2+}$  solution on the  $\text{Ca}^{2+}$  concentration in the  $\text{Ca}^{2+}$  solution. The  $\text{Ca}^{2+}$  concentration in the  $\text{Ca}^{2+}$  solution was 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4.0, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 5.0, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 6.0, 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 7.0, 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.8, 7.9, 8.0, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8, 8.9, 9.0, 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 10.0, 10.1, 10.2, 10.3, 10.4, 10.5, 10.6, 10.7, 10.8, 10.9, 11.0, 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 12.0, 12.1, 12.2, 12.3, 12.4, 12.5, 12.6, 12.7, 12.8, 12.9, 13.0, 13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.8, 13.9, 14.0, 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9, 15.0, 15.1, 15.2, 15.3, 15.4, 15.5, 15.6, 15.7, 15.8, 15.9, 16.0, 16.1, 16.2, 16.3, 16.4, 16.5, 16.6, 16.7, 16.8, 16.9, 17.0, 17.1, 17.2, 17.3, 17.4, 17.5, 17.6, 17.7, 17.8, 17.9, 18.0, 18.1, 18.2, 18.3, 18.4, 18.5, 18.6, 18.7, 18.8, 18.9, 19.0, 19.1, 19.2, 19.3, 19.4, 19.5, 19.6, 19.7, 19.8, 19.9, 20.0, 20.1, 20.2, 20.3, 20.4, 20.5, 20.6, 20.7, 20.8, 20.9, 21.0, 21.1, 21.2, 21.3, 21.4, 21.5, 21.6, 21.7, 21.8, 21.9, 22.0, 22.1, 22.2, 22.3, 22.4, 22.5, 22.6, 22.7, 22.8, 22.9, 23.0, 23.1, 23.2, 23.3, 23.4, 23.5, 23.6, 23.7, 23.8, 23.9, 24.0, 24.1, 24.2, 24.3, 24.4, 24.5, 24.6, 24.7, 24.8, 24.9, 25.0, 25.1, 25.2, 25.3, 25.4, 25.5, 25.6, 25.7, 25.8, 25.9, 26.0, 26.1, 26.2, 26.3, 26.4, 26.5, 26.6, 26.7, 26.8, 26.9, 27.0, 27.1, 27.2, 27.3, 27.4, 27.5, 27.6, 27.7, 27.8, 27.9, 28.0, 28.1, 28.2, 28.3, 28.4, 28.5, 28.6, 28.7, 28.8, 28.9, 29.0, 29.1, 29.2, 29.3, 29.4, 29.5, 29.6, 29.7, 29.8, 29.9, 30.0, 30.1, 30.2, 30.3, 30.4, 30.5, 30.6, 30.7, 30.8, 30.9, 31.0, 31.1, 31.2, 31.3, 31.4, 31.5, 31.6, 31.7, 31.8, 31.9, 32.0, 32.1, 32.2, 32.3, 32.4, 32.5, 32.6, 32.7, 32.8, 32.9, 33.0, 33.1, 33.2, 33.3, 33.4, 33.5, 33.6, 33.7, 33.8, 33.9, 34.0, 34.1, 34.2, 34.3, 34.4, 34.5, 34.6, 34.7, 34.8, 34.9, 35.0, 35.1, 35.2, 35.3, 35.4, 35.5, 35.6, 35.7, 35.8, 35.9, 36.0, 36.1, 36.2, 36.3, 36.4, 36.5, 36.6, 36.7, 36.8, 36.9, 37.0, 37.1, 37.2, 37.3, 37.4, 37.5, 37.6, 37.7, 37.8, 37.9, 38.0, 38.1, 38.2, 38.3, 38.4, 38.5, 38.6, 38.7, 38.8, 38.9, 39.0, 39.1, 39.2, 39.3, 39.4, 39.5, 39.6, 39.7, 39.8, 39.9, 40.0, 40.1, 40.2, 40.3, 40.4, 40.5, 40.6, 40.7, 40.8, 40.9, 41.0, 41.1, 41.2, 41.3, 41.4, 41.5, 41.6, 41.7, 41.8, 41.9, 42.0, 42.1, 42.2, 42.3, 42.4, 42.5, 42.6, 42.7, 42.8, 42.9, 43.0, 43.1, 43.2, 43.3, 43.4, 43.5, 43.6, 43.7, 43.8, 43.9, 44.0, 44.1, 44.2, 44.3, 44.4, 44.5, 44.6, 44.7, 44.8, 44.9, 45.0, 45.1, 45.2, 45.3, 45.4, 45.5, 45.6, 45.7, 45.8, 45.9, 46.0, 46.1, 46.2, 46.3, 46.4, 46.5, 46.6, 46.7, 46.8, 46.9, 47.0, 47.1, 47.2, 47.3, 47.4, 47.5, 47.6, 47.7, 47.8, 47.9, 48.0, 48.1, 48.2, 48.3, 48.4, 48.5, 48.6, 48.7, 48.8, 48.9, 49.0, 49.1, 49.2, 49.3, 49.4, 49.5, 49.6, 49.7, 49.8, 49.9, 50.0, 50.1, 50.2, 50.3, 50.4, 50.5, 50.6, 50.7, 50.8, 50.9, 51.0, 51.1, 51.2, 51.3, 51.4, 51.5, 51.6, 51.7, 51.8, 51.9, 52.0, 52.1, 52.2, 52.3, 52.4, 52.5, 52.6, 52.7, 52.8, 52.9, 53.0, 53.1, 53.2, 53.3, 53.4, 53.5, 53.6, 53.7, 53.8, 53.9, 54.0, 54.1, 54.2, 54.3, 54.4, 54.5, 54.6, 54.7, 54.8, 54.9, 55.0, 55.1, 55.2, 55.3, 55.4, 55.5, 55.6, 55.7, 55.8, 55.9, 56.0, 56.1, 56.2, 56.3, 56.4, 56.5, 56.6, 56.7, 56.8, 56.9, 57.0, 57.1, 57.2, 57.3, 57.4, 57.5, 57.6, 57.7, 57.8, 57.9, 58.0, 58.1, 58.2, 58.3, 58.4, 58.5, 58.6, 58.7, 58.8, 58.9, 59.0, 59.1, 59.2, 59.3, 59.4, 59.5, 59.6, 59.7, 59.8, 59.9, 60.0, 60.1, 60.2, 60.3, 60.4, 60.5, 60.6, 60.7, 60.8, 60.9, 61.0, 61.1, 61.2, 61.3, 61.4, 61.5, 61.6, 61.7, 61.8, 61.9, 62.0, 62.1, 62.2, 62.3, 62.4, 62.5, 62.6, 62.7, 62.8, 62.9, 63.0, 63.1, 63.2, 63.3, 63.4, 63.5, 63.6, 63.7, 63.8, 63.9, 64.0, 64.1, 64.2, 64.3, 64.4, 64.5, 64.6, 64.7, 64.8, 64.9, 65.0, 65.1, 65.2, 65.3, 65.4, 65.5, 65.6, 65.7, 65.8, 65.9, 66.0, 66.1, 66.2, 66.3, 66.4, 66.5, 66.6, 66.7, 66.8, 66.9, 67.0, 67.1, 67.2, 67.3, 67.4, 67.5, 67.6, 67.7, 67.8, 67.9, 68.0, 68.1, 68.2, 68.3, 68.4

*[Faint, illegible handwritten notes]*

the following information is being furnished to you for your information only and is not to be used for any other purpose.

FILED IN: 62-108977-1000

ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED

~~ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED~~

no defective or not used material from 1948 to 1951. The only one of these

1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 26

100

T. T. 47 T. 10

Figure 1

... ..

1952年12月15日

REF ID: A66049

THE UNIVERSITY OF CHICAGO LIBRARY

1961-1962

...and the ... ..

[illegible]

system, which is a variable-speed optical analyzer employing a resonant vibration galvanometer. Therefore, it is not surprising that the eventual solutions contained in Equations 34 and 36 are of the same form as the results obtained by Barber and Ursell. These authors have plotted envelopes of this transient resonance.<sup>2,3</sup> Their results, with appropriate changes in notation, are presented in Figure 2.4. Similar results would be obtained by use of any linear, second-order system. Universal resonance curves are made available which can be applied to a very good approximation. These are useful since many complex selective networks can be approximated by a linear second-order system. For example, Scanes<sup>19</sup> describes a parallel-T network which is exactly equivalent to a series ICR circuit.

Curve a of Figure 2.4 is the characteristic steady-state response with the peak arbitrarily set at zero decibels. For curves b through g the amplitudes of the peak input signal would be underestimated by the indicated power error. A simple relationship exists between this dynamic depression error and the parameter  $K$ .<sup>\*</sup> Also note that the peak transmission always occurs after the instant when the excitation frequency equal  $f_p$ . This frequency displacement distortion is also related to the parameter  $K$ .<sup>\*\*</sup> Study of Figure 2.4 indicates that the effective bandwidth,  $(\Delta f)_e$ , is greater than the nominal band-width,  $\Delta f$ .<sup>\*\*\*</sup> Furthermore, secondary peaks are observed which could mask the detection of weak input components.

Note that Figure 2.4 does not take into account two important aspects of the general problem which should be considered for the proposed analyzer.

---

\* See Figure 3.3

\* See Figure 3.4 based on several sources including Figure 2.4

\* See Figure 3.5

1. The first part of the report deals with the general situation of the country.

2. The second part of the report deals with the economic situation of the country.

3. The third part of the report deals with the social situation of the country.

4. The fourth part of the report deals with the political situation of the country.

5. The fifth part of the report deals with the cultural situation of the country.

6. The sixth part of the report deals with the environmental situation of the country.

7. The seventh part of the report deals with the international situation of the country.

8. The eighth part of the report deals with the future prospects of the country.

9. The ninth part of the report deals with the conclusion of the report.

10. The tenth part of the report deals with the annexes of the report.

11. The eleventh part of the report deals with the bibliography of the report.

12. The twelfth part of the report deals with the index of the report.

13. The thirteenth part of the report deals with the list of figures of the report.

14. The fourteenth part of the report deals with the list of tables of the report.

15. The fifteenth part of the report deals with the list of maps of the report.

16. The sixteenth part of the report deals with the list of abbreviations of the report.

17. The seventeenth part of the report deals with the list of symbols of the report.

18. The eighteenth part of the report deals with the list of units of the report.

19. The nineteenth part of the report deals with the list of references of the report.

20. The twentieth part of the report deals with the list of footnotes of the report.

21. The twenty-first part of the report deals with the list of appendices of the report.

22. The twenty-second part of the report deals with the list of annexes of the report.

23. The twenty-third part of the report deals with the list of bibliographies of the report.

24. The twenty-fourth part of the report deals with the list of indexes of the report.

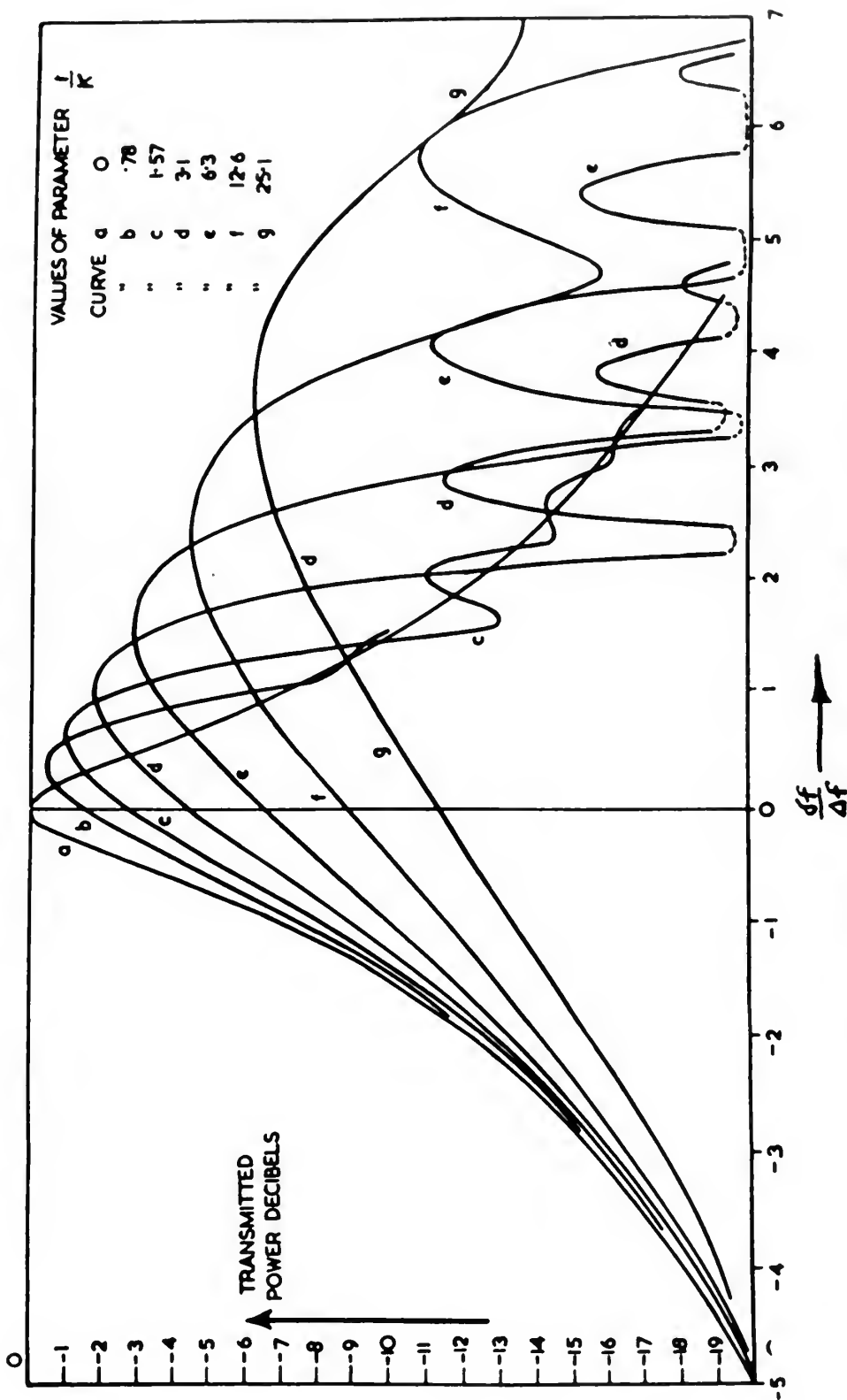
25. The twenty-fifth part of the report deals with the list of figures of the report.

26. The twenty-sixth part of the report deals with the list of tables of the report.

27. The twenty-seventh part of the report deals with the list of maps of the report.



FIGURE 2.4  
TRANSMISSION OF A SIMPLE SELECTIVE  
NETWORK IN CONTINUOUS FREQUENCY ANALYSIS



[ Source: Reference 3 and Thesis Equation 36]



The first occurs when initial conditions are not zero. Marique<sup>13</sup> and Soanes<sup>19,20</sup> each discuss the additional transient term which exists in the response. The second aspect is the determination of the effect of starting the sweep in or near the pass-band. Soanes<sup>19</sup> presents a solution to this problem by an extension of the work of Barber and Ursell. Instead of building up to the maximum response in a smooth fashion as indicated in Figure 2.4, "oscillations" occur near the beginning of the sweep.

In conclusion, six different distortions can exist:

(1) amplitude depression, (2) frequency displacement, (3) decreased selectivity,  $Q$ , (4) secondary peak masking (ringing), (5) initial-conditions transients and (6) uneven leading edges resulting from starting the sweep in or near the pass-band.

#### Magnetic Recording Characteristics

Modern tape is sufficiently constant in sensitivity to allow amplitude recording to a few percent.<sup>24</sup> It has been shown that there exist workable analyzers which employ a magnetic medium. Hence, magnetic properties and performance characteristics of recording tapes do not in themselves determine the feasibility of the proposed analyzer. Yet, at least three of these characteristics must be recognized as major factors in the overall design.

First, there is a distinct advantage in continuous operation at a reproduce speed which is always greater than the constant record speed. Use of multiplication rather than division results in increased output magnitudes and increased signal-to-noise ratios. However, in order to keep the number of selective networks and graphic recorder channels to a minimum it may be necessary to partially speed-up and partially slow-

The first part of the report is devoted to a description of the general situation in the country. It is followed by a detailed analysis of the economic situation, which shows a steady decline in the standard of living of the population. The third part of the report is devoted to a description of the political situation, which is characterized by a lack of democracy and freedom of expression. The fourth part of the report is devoted to a description of the social situation, which is characterized by a high level of unemployment and a lack of social security.

The fifth part of the report is devoted to a description of the cultural situation, which is characterized by a lack of freedom of expression and a high level of censorship. The sixth part of the report is devoted to a description of the environmental situation, which is characterized by a high level of pollution and a lack of environmental protection. The seventh part of the report is devoted to a description of the international situation, which is characterized by a lack of cooperation and a high level of tension.

### CONCLUSIONS

The report shows that the country is in a state of deep crisis. The economic situation is dire, the political situation is undemocratic, the social situation is deplorable, the cultural situation is repressive, the environmental situation is disastrous, and the international situation is tense. The only way out of this crisis is through a complete and radical reform of the country's political, economic, social, cultural, environmental, and international relations.

The report also shows that the population is suffering from a lack of basic human rights and freedoms. The government is responsible for this situation and must be held accountable for its actions. The population has the right to a better life and a more democratic and free society. The report calls for a complete and radical reform of the country's political, economic, social, cultural, environmental, and international relations.

down (as compared to the fixed recorded speed) the reproduce tape for a wide frequency-band analysis. Secondly, a definite limitation occurs because of imperfect magnetic contact between the reproducing head and the recording medium. A rigid requirement for this analyzer is that it provides constant contact of the playback head against the magnetic tape. R. L. Wallace, Jr. of the Bell Laboratories<sup>31</sup> has made experimental and theoretical determinations of the spacing losses involved when the reproduce head loses contact with the magnetic surface. He reports that

$$\text{Spacing loss (db)} = 55 (d/\lambda)$$

where

d = spacing introduced between reproducing head and magnetic medium

$\lambda = \text{recorded wavelength} = \frac{S_r}{f_{an}}$

db = decibels that reproduced voltage level is decreased

It is to be expected that the magnetic contact between the reproduce head and the medium is less than perfect. Modulation noise introduces a spacing loss for devices exhibiting only apparent intimate contact. For example, imperfect magnetic contact can result from chattering of the tape on the reproduce head, or from changes in the degree of contact due to clumps on the tape surface.<sup>23</sup> Amplitude modulation of the reproduced signal results. Specifically, let us consider the case when a non-magnetic disc with tape wrapped around the circumference is used as the multiplier. Any eccentricity in the motion of the disc will introduce the amplitude modulation described above. Furthermore, this eccentricity will result in excessive wear in the heads, which increases the area in contact with the tape and decreases the signal-to-noise ratio.

Lastly, a characteristic high frequency loss due to the length of the playback head gap will probably determine the upper frequency limit



for the proposed analyzer. When the reproduce wave length ( $\lambda = \frac{S}{f_{ao}}$ ) approaches the effective playback gap length,  $\beta$ , a decreased signal output will occur. Lennert<sup>27</sup> predicts that this loss can be expressed as

$$\text{Voltage level loss} = 20 \log_{10} \frac{\sin \frac{\pi \beta}{\lambda}}{\frac{\pi \beta}{\lambda}}$$

Hence, there exists a certain minimum wave length the reproduce head will recognize.

### Sample Length Considerations

Appendix D consists of an investigation on sample length requirements.

The results can be summarized as

$$L \text{ (seconds)} = \frac{2 K f_u}{f_{ao} \Delta f} \quad (39)$$

$$L \text{ (cycles)} = \frac{2 K f_u}{\Delta f} \quad (39a)$$

where

$L$  = recorded sample length

### Summary

Six key questions regarding the proposed analyzer were raised at the end of Chapter I. A basic theory has been formulated in Chapter II-A, and in Appendices B, C, and D which provides a basis for answering these questions.





## CHAPTER II-B

### EXPERIMENTAL PROCEDURE

#### Introduction

Figure 1.1 showed the functional operations necessary to be performed on an unknown recorded input in order to extract the desired spectral density of that input. Figure 2.5 shows in block diagram form the arrangement of components which was used to accomplish the aforementioned functions. The following discussion of these components refers to those actually used. A more thorough discussion of major components to accomplish the desired functions is given in Appendix E.

#### Outline of Operation of Block Diagram

The unknown sample to be analyzed is taken as the starting point for the block diagram. This tape is wrapped on the periphery of circular disc 9.00 inches in diameter. A reproduce head is mounted so that the magnetic tape on the disc revolves past the face of the reproduce head. The movement of the tape past the gap of the reproduce head causes a voltage to be generated in the windings of the head in accordance with the signal on the tape as modified by the linear speed of transition of the tape past the gap. Because the voltage output of the reproduce head is rather low (0.004 volts maximum for the heads used, provided that the tape is in contact with the face of the head) a voltage amplifier is used to raise the level of this out-put. The output of the amplifier may be composed of many different frequencies depending on the character of the recorded input. The selective system picks out one frequency component so that this component can be measured. The selective system for this arrangement is a high Q parallel resonant tuned circuit employing positive feedback for Q multiplication. The output of the tuned circuit is fed through

... is given in Appendix A.

Outline of Operation of the Recorder

The recorder consists of a magnetic tape and a magnetic head. The magnetic tape is a continuous strip of material on which the magnetic flux is recorded. The magnetic head is a device which is used to read the magnetic flux from the tape. The recorder is operated by a control unit which is connected to the magnetic tape and the magnetic head. The control unit is responsible for the operation of the recorder and for the recording of the magnetic flux on the tape. The recorder is used to record the magnetic flux from a source and to reproduce it later. The recorder is a very important device in the field of magnetic recording.

a cathode follower isolation stage to a conventional linear detector. The detector determines the amplitude of the single frequency output of the tuned circuit as a function of time, and this amplitude, varying at a much slower rate than the undetected output of the resonant circuit, is capable of being recorded permanently on a low-frequency, paper recorder.

The speed of the disc carrying the magnetic tape is controlled by a Ward-Leonard type of speed control. (However, this control system was run open loop using the typical speed characteristics of an armature controlled, direct current motor rather than closed loop with tachometer feedback as originally planned) The controlling signal to the Ward-Leonard system is developed in a function generator. Only an exponential variation was investigated and this voltage was developed in a simple RC exponential decay circuit. The DC output of the function generator is amplified by the voltage amplifier and the power amplifier controls the field current of a DC generator whose armature output is connected in series with the armature of a DC motor. The field of this motor is supplied from an external constant source. Thus the combined system provides an armature controlled DC motor with which the speed of the disc might be controlled. The controlling voltage and the speed of the motor, indicated by the output voltage of a tachometer, are both monitored by paper recorders. The input to the frequency selective tuned circuit is monitored by a cathode ray oscilloscope.

It was determined that the driving arrangement for this system resulted in an uneven variation of frequency as the played-back frequency of a recorded component was swept across the tuned circuit. In order to achieve a smooth variation of excitation frequency for the tuned circuit, the disc was gotten up to the desired speed and then was permitted to slow down by

The system is designed to provide a means of controlling the speed of a motor. The speed of the motor is controlled by a feedback system which compares the actual speed of the motor with a desired speed. The feedback system is a closed-loop system which uses a feedback signal to adjust the motor speed. The feedback signal is obtained from a speed sensor which measures the actual speed of the motor. The feedback signal is compared with a reference speed to produce an error signal. The error signal is then used to adjust the motor speed. The feedback system is designed to provide a means of controlling the speed of a motor. The speed of the motor is controlled by a feedback system which compares the actual speed of the motor with a desired speed. The feedback system is a closed-loop system which uses a feedback signal to adjust the motor speed. The feedback signal is obtained from a speed sensor which measures the actual speed of the motor. The feedback signal is compared with a reference speed to produce an error signal. The error signal is then used to adjust the motor speed.

itself. The butt joint transient of the recorded sample was used as a marker in order to indicate the variation in speed of the free running disc. This transient was recorded along with the response of the tuned circuit on a twin paper recorder. The revolutions of the disc as indicated by the butt joint markers were calibrated in time by using one second markers which were imposed on a third trace of the paper recorder. A block diagram of this arrangement is given in Figure 2.6.

1. The first of these is the fact that the majority of the population of the United States is of European descent. This is true of the United States as a whole, and also of the individual States. The majority of the population of the United States is of European descent, and this is true of the individual States. The majority of the population of the United States is of European descent, and this is true of the individual States.

FIGURE 2.5  
Block Diagram Driven System

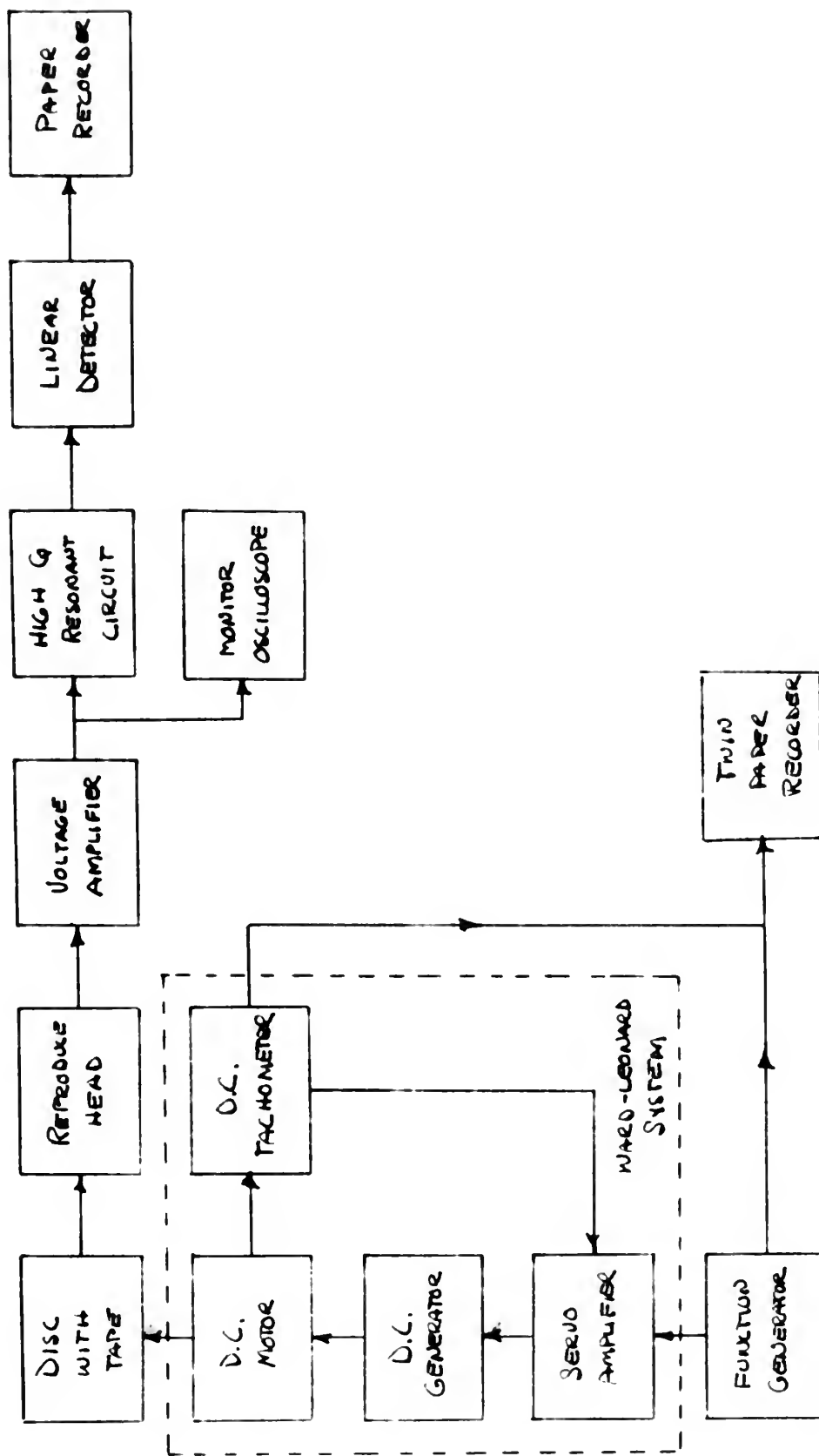
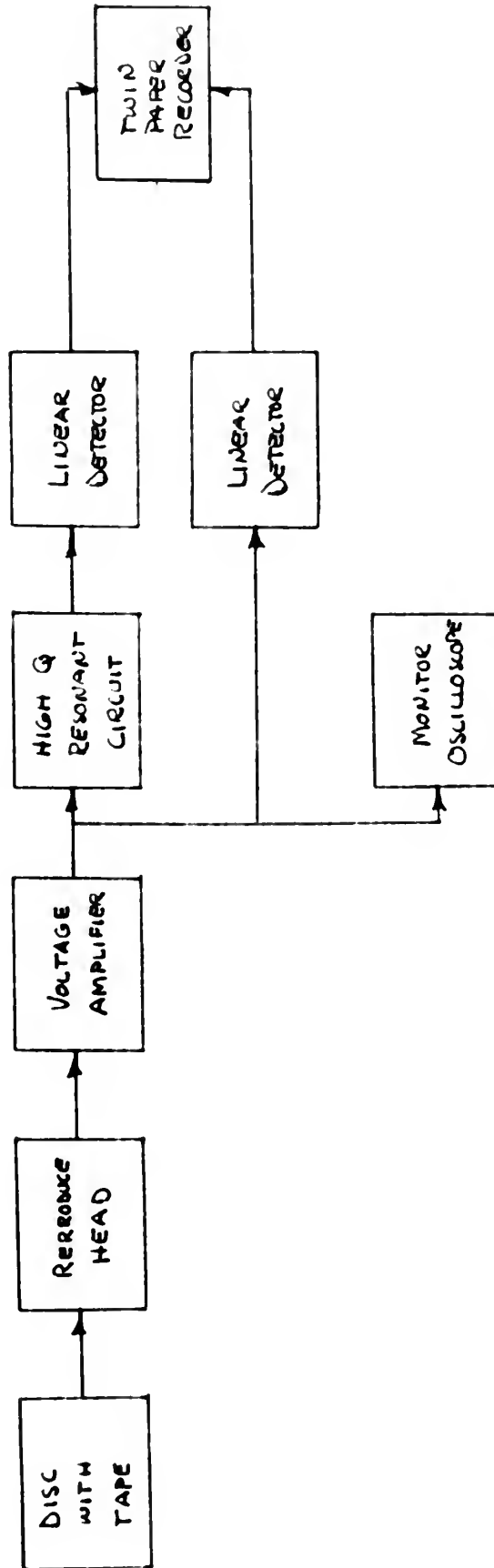






FIGURE 2.6

Block Diagram - Free Running  
System





## CHAPTER III

### RESULTS

#### Theoretical Results

A basic analyzer theory has been formulated in Chapter II-A, and in Appendices B, C, and D. The following results of this investigation are of specific interest:

(1) Two multiplier speed-time relationships were determined and are presented in Figures 3.1 and 3.2. For the minimum-analysis-time solution each analyzed component remains within the pass-band of the selective network the minimum time for acceptable analysis. For the equal-sample-analysis solution each analyzed component remains under scrutiny for the entire length of sample.

(2) The term acceptable analysis is based on having the output of the selective network remain within tolerable limits of amplitude and frequency distortion which are imposed by the characteristics of the selective network. Figures 3.3, 3.4, and 3.5 predict this distortion as a function of a parameter K. This constant relates the characteristics of the excitation frequency with those of the selective network. Note that these results are applicable only for the range of K indicated in each figure.

(3) The recorded sample of tape selected for analysis must have a certain minimum length

$$L = \frac{2 K f_u}{f_{ao} \Delta f}$$

where

L = length of sample measured in seconds.

$f_u$  = the upper cut-off frequency of the selective network.

$\Delta f$  = the band-width of the selective network.

ANALYSIS OF THE RESULTS

A main analysis has been performed in Chapter II-A, and in Appendices B, C, and D. The following results of this investigation are of specific interest:

(1) The multiplier speed-time relationships were determined and are presented in Figures 1.1 and 1.2. For the minimum-analysis-time solution each analyzed component remains within the pass-band of the selective network the minimum time for acceptable analysis. For the equal-sample-analysis solution each analyzed component remains within the pass-band of the selective network for the entire length of sample.

(2) The term acceptable analysis is based on having the output of the selective network remain within tolerable limits of amplitude and frequency distortion which are imposed by the characteristics of the selective network. Figures 1.3, 1.4, and 1.5 predict this distortion as a function of a parameter K. This constant relates the characteristics of the excitation frequency with those of the selective network. Note that these results are applicable only for the range of K indicated in each figure.

(3) The recorded sample of tape selected for analysis must have a

$$\begin{aligned} \text{certain minimum length} \\ L \geq \frac{1}{\Delta f} \\ \text{where} \end{aligned}$$

where

$L$  = length of sample (seconds)

$\Delta f$  = frequency resolution of the selective network

$\Delta f$  = the bandwidth of the selective network

$f_{ao}$  = the lowest recorded frequency component being analyzed.

$$K = \frac{(\Delta f)^2}{\frac{df_{bn}}{dt}} \text{ cycles}$$

$f_{bn}$  = the output of the frequency multiplier

### Experimental Results

Two factors have combined to make the experimental results largely qualitative rather than quantitative. These are

- (1) Variation of the amplitude of the reproduced signal with angular position of the disc due to the eccentricity of the disc.
- (2) Inability to control the speed of the disc within narrow limits

Figure 3.6 shows in a qualitative manner the analysis of a composite recorded sample. The recorded frequencies and the analyzed frequencies are given below. It is important to note that the recorded sample has not been analyzed by an independent means.

<u>Recording Frequencies</u>		<u>Analyzed Frequencies</u>	
Frequency	Relative Amplitude	Frequency	Relative Amplitude
		184	.71
		188	1.56
198	1.00	198	1.00
393	1.00	392	.71
		408	.36
418	1.00	418	.93
		618	4.42
686	.50		
696	1.00	694	.71
		842	.20
886	1.00		
891	.50		
1180	1.00		

Figure 1.1 shows the magnitude of the recorded signal

# Experimental Results

Two factors have combined to make the experimental results largely

qualitative rather than quantitative. These are:

(1) Variation of the magnitude of the recorded

signal with angular position of the disc due

to the eccentricity of the disc.

(2) Difficulty to control the speed of the disc

within narrow limits

Figure 1.2 shows a qualitative summary of the analysis of a com-

parison recorded sample. The recorded frequencies and the angular frequency

are given below. It is important to note that the recorded sample

has not been analysed by an independent means.

## Recorded Frequencies

## Angular Frequency

Frequency	Relative Amplitude	Frequency	Relative Amplitude
1.00	1.00	1.00	1.00
1.05	1.05	1.05	1.05
1.10	1.10	1.10	1.10
1.15	1.15	1.15	1.15
1.20	1.20	1.20	1.20
1.25	1.25	1.25	1.25
1.30	1.30	1.30	1.30
1.35	1.35	1.35	1.35
1.40	1.40	1.40	1.40
1.45	1.45	1.45	1.45
1.50	1.50	1.50	1.50
1.55	1.55	1.55	1.55
1.60	1.60	1.60	1.60
1.65	1.65	1.65	1.65
1.70	1.70	1.70	1.70
1.75	1.75	1.75	1.75
1.80	1.80	1.80	1.80
1.85	1.85	1.85	1.85
1.90	1.90	1.90	1.90
1.95	1.95	1.95	1.95
2.00	2.00	2.00	2.00

The disc for the above analysis was permitted to slow down freely. The variation of the linear speed of the tape is given in figure 3.8.

Figure 3.7 gives a measure of the resolution of the analyzer for a  $Q$  of the tuned circuit of 625, and a resonant frequency of 5020. The speed-time relationship is similar to that given in Figure 3.8 and results in a parameter of  $K$  equal to 2.46 at 400 cycles per second and a value of  $K$  equal to 1.47 at 700 cycles per second.

The shape of the output pulse for one recorded frequency, 300 cycles per second is given in Figure 3.9 for different values of the parameter  $K$ .

[illegible]

To find out more about us or our products please visit our website at [www.dunlop.com](http://www.dunlop.com)

sent-increase will 1972 to 1974-1975 1976-1977 1978-1979 1980-1981 1982-1983 1984-1985 1986-1987 1988-1989 1990-1991 1992-1993 1994-1995 1996-1997 1998-1999 2000-2001 2002-2003 2004-2005 2006-2007 2008-2009 2010-2011 2012-2013 2014-2015 2016-2017 2018-2019 2020-2021 2022-2023 2024-2025 2026-2027 2028-2029 2030-2031 2032-2033 2034-2035 2036-2037 2038-2039 2040-2041 2042-2043 2044-2045 2046-2047 2048-2049 2050-2051 2052-2053 2054-2055 2056-2057 2058-2059 2060-2061 2062-2063 2064-2065 2066-2067 2068-2069 2070-2071 2072-2073 2074-2075 2076-2077 2078-2079 2080-2081 2082-2083 2084-2085 2086-2087 2088-2089 2090-2091 2092-2093 2094-2095 2096-2097 2098-2099 2100-2101 2102-2103 2104-2105 2106-2107 2108-2109 2110-2111 2112-2113 2114-2115 2116-2117 2118-2119 2120-2121 2122-2123 2124-2125 2126-2127 2128-2129 2130-2131 2132-2133 2134-2135 2136-2137 2138-2139 2140-2141 2142-2143 2144-2145 2146-2147 2148-2149 2150-2151 2152-2153 2154-2155 2156-2157 2158-2159 2160-2161 2162-2163 2164-2165 2166-2167 2168-2169 2170-2171 2172-2173 2174-2175 2176-2177 2178-2179 2180-2181 2182-2183 2184-2185 2186-2187 2188-2189 2190-2191 2192-2193 2194-2195 2196-2197 2198-2199 2200-2201 2202-2203 2204-2205 2206-2207 2208-2209 2210-2211 2212-2213 2214-2215 2216-2217 2218-2219 2220-2221 2222-2223 2224-2225 2226-2227 2228-2229 2230-2231 2232-2233 2234-2235 2236-2237 2238-2239 2240-2241 2242-2243 2244-2245 2246-2247 2248-2249 2250-2251 2252-2253 2254-2255 2256-2257 2258-2259 2260-2261 2262-2263 2264-2265 2266-2267 2268-2269 2270-2271 2272-2273 2274-2275 2276-2277 2278-2279 2280-2281 2282-2283 2284-2285 2286-2287 2288-2289 2290-2291 2292-2293 2294-2295 2296-2297 2298-2299 2300-2301 2302-2303 2304-2305 2306-2307 2308-2309 2310-2311 2312-2313 2314-2315 2316-2317 2318-2319 2320-2321 2322-2323 2324-2325 2326-2327 2328-2329 2330-2331 2332-2333 2334-2335 2336-2337 2338-2339 2340-2341 2342-2343 2344-2345 2346-2347 2348-2349 2350-2351 2352-2353 2354-2355 2356-2357 2358-2359 2360-2361 2362-2363 2364-2365 2366-2367 2368-2369 2370-2371 2372-2373 2374-2375 2376-2377 2378-2379 2380-2381 2382-2383 2384-2385 2386-2387 2388-2389 2390-2391 2392-2393 2394-2395 2396-2397 2398-2399 2400-2401 2402-2403 2404-2405 2406-2407 2408-2409 2410-2411 2412-2413 2414-2415 2416-2417 2418-2419 2420-2421 2422-2423 2424-2425 2426-2427 2428-2429 2430-2431 2432-2433 2434-2435 2436-2437 2438-2439 2440-2441 2442-2443 2444-2445 2446-2447 2448-2449 2450-2451 2452-2453 2454-2455 2456-2457 2458-2459 2460-2461 2462-2463 2464-2465 2466-2467 2468-2469 2470-2471 2472-2473 2474-2475 2476-2477 2478-2479 2480-2481 2482-2483 2484-2485 2486-2487 2488-2489 2490-2491 2492-2493 2494-2495 2496-2497 2498-2499 2500-2501 2502-2503 2504-2505 2506-2507 2508-2509 2510-2511 2512-2513 2514-2515 2516-2517 2518-2519 2520-2521 2522-2523 2524-2525 2526-2527 2528-2529 2530-2531 2532-2533 2534-2535 2536-2537 2538-2539 2540-2541 2542-2543 2544-2545 2546-2547 2548-2549 2550-2551 2552-2553 2554-2555 2556-2557 2558-2559 2560-2561 2562-2563 2564-2565 2566-2567 2568-2569 2570-2571 2572-2573 2574-2575 2576-2577 2578-2579 2580-2581 2582-2583 2584-2585 2586-2587 2588-2589 2590-2591 2592-2593 2594-2595 2596-2597 2598-2599 2600-2601 2602-2603 2604-2605 2606-2607 2608-2609 2610-2611 2612-2613 2614-2615 2616-2617 2618-2619 2620-2621 2622-2623 2624-2625 2626-2627 2628-2629 2630-2631 2632-2633 2634-2635 2636-2637 2638-2639 2640-2641 2642-2643 2644-2645 2646-2647 2648-2649 2650-2651 2652-2653 2654-2655 2656-2657 2658-2659 2660-2661 2662-2663 2664-2665 2666-2667 2668-2669 2670-2671 2672-2673 2674-2675 2676-2677 2678-2679 2680-2681 2682-2683 2684-2685 2686-2687 2688-2689 2690-2691 2692-2693 2694-2695 2696-2697 2698-2699 2700-2701 2702-2703 2704-2705 2706-2707 2708-2709 2710-2711 2712-2713 2714-2715 2716-2717 2718-2719 2720-2721 2722-2723 2724-2725 2726-2727 2728-2729 2730-2731 2732-2733 2734-2735 2736-2737 2738-2739 2740-2741 2742-2743 2744-2745 2746-2747 2748-2749 2750-2751 2752-2753 2754-2755 2756-2757 2758-2759 2760-2761 2762-2763 2764-2765 2766-2767 2768-2769 2770-2771 2772-2773 2774-2775 2776-2777 2778-2779 2780-2781 2782-2783 2784-2785 2786-2787 2788-2789 2790-

[illegible]

Large I to allow a hot burner run since the 1000 W burner is not

*Journal of Management Studies*, 19(1), 67-80.

bioRxiv preprint doi: <https://doi.org/10.1101/000000>; this version posted April 1, 2014. The copyright holder for this preprint (which was not certified by peer review) is the author/funder, who has granted bioRxiv a license to display the preprint in perpetuity. It is made available under aCC-BY-NC-ND 4.0 International license.

4. Information on the number of persons who are not registered as voters in the state of New York is as follows:



FIGURE 3.1

MULTIPLIER SPEED-TIME RELATIONSHIPS, I

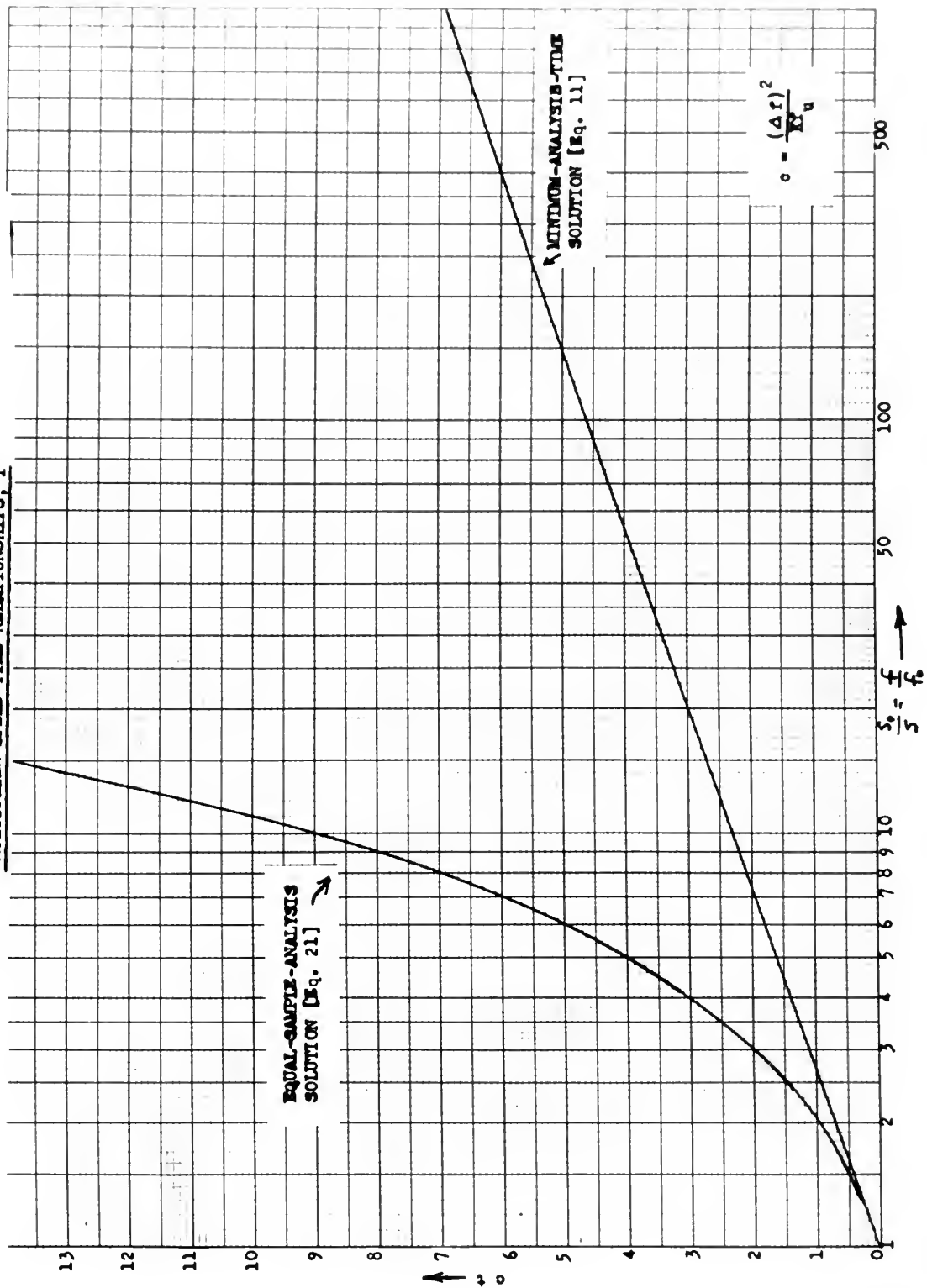




FIGURE 3.2

MULTIPLIER SPEED-TIME RELATIONSHIPS, II

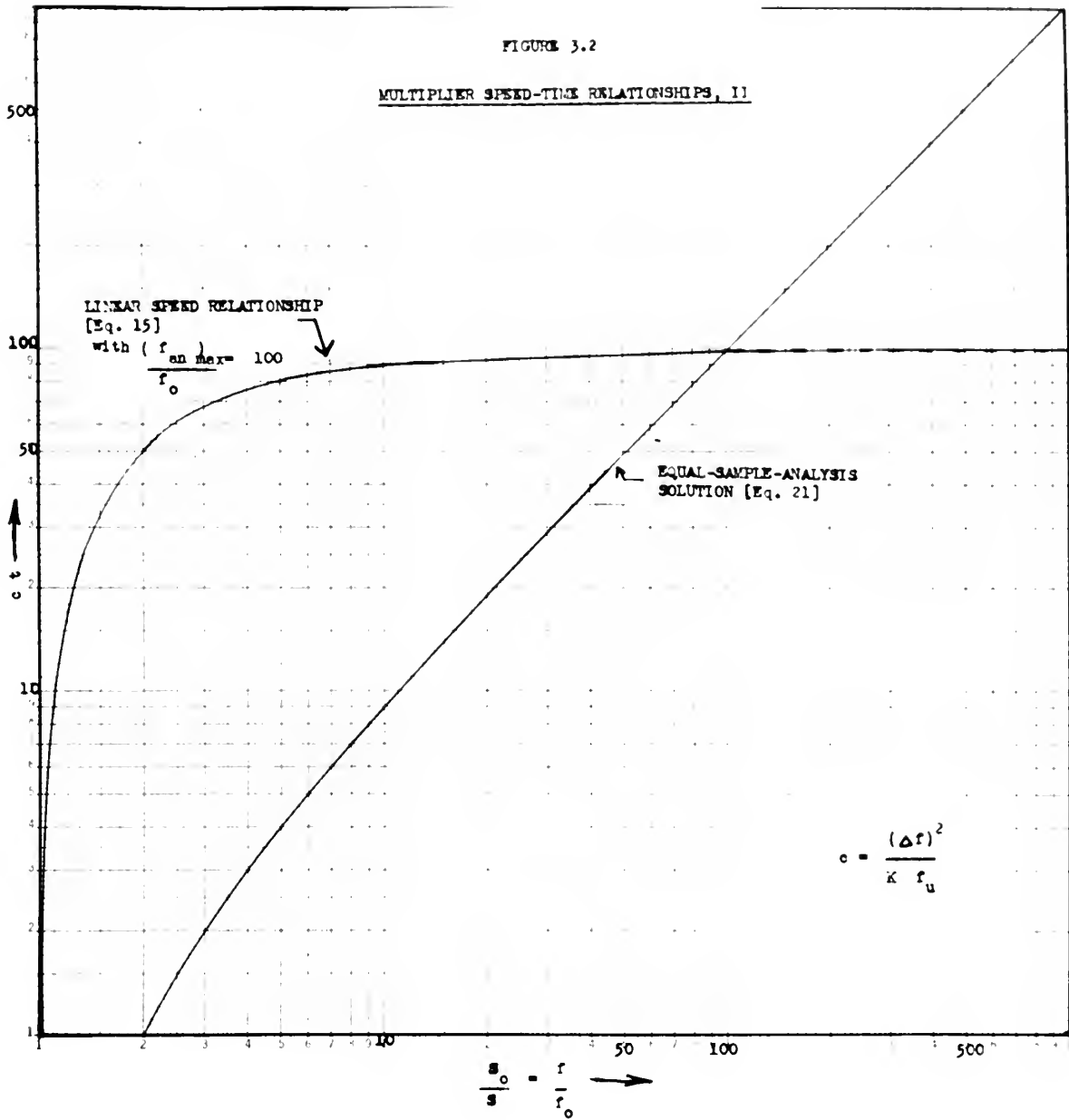




FIGURE 3.3  
PEAK TRANSMISSION POWER ERROR FOR  
CONTINUOUS ANALYSIS OF SIMPLE SELECTIVE NETWORK

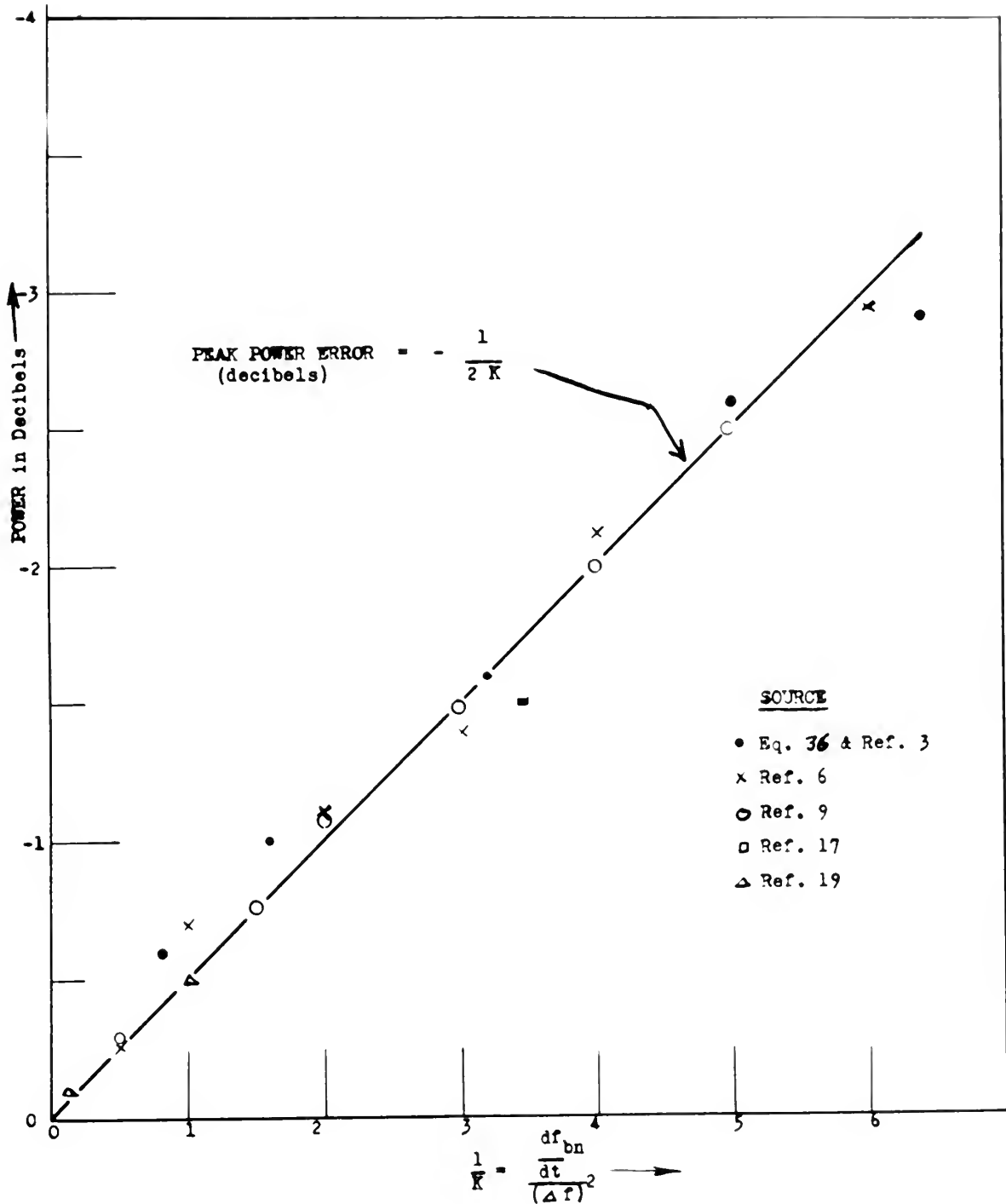




FIGURE 3.4

FREQUENCY LAG IN PEAK TRANSMISSION FOR CONTINUOUS ANALYSIS

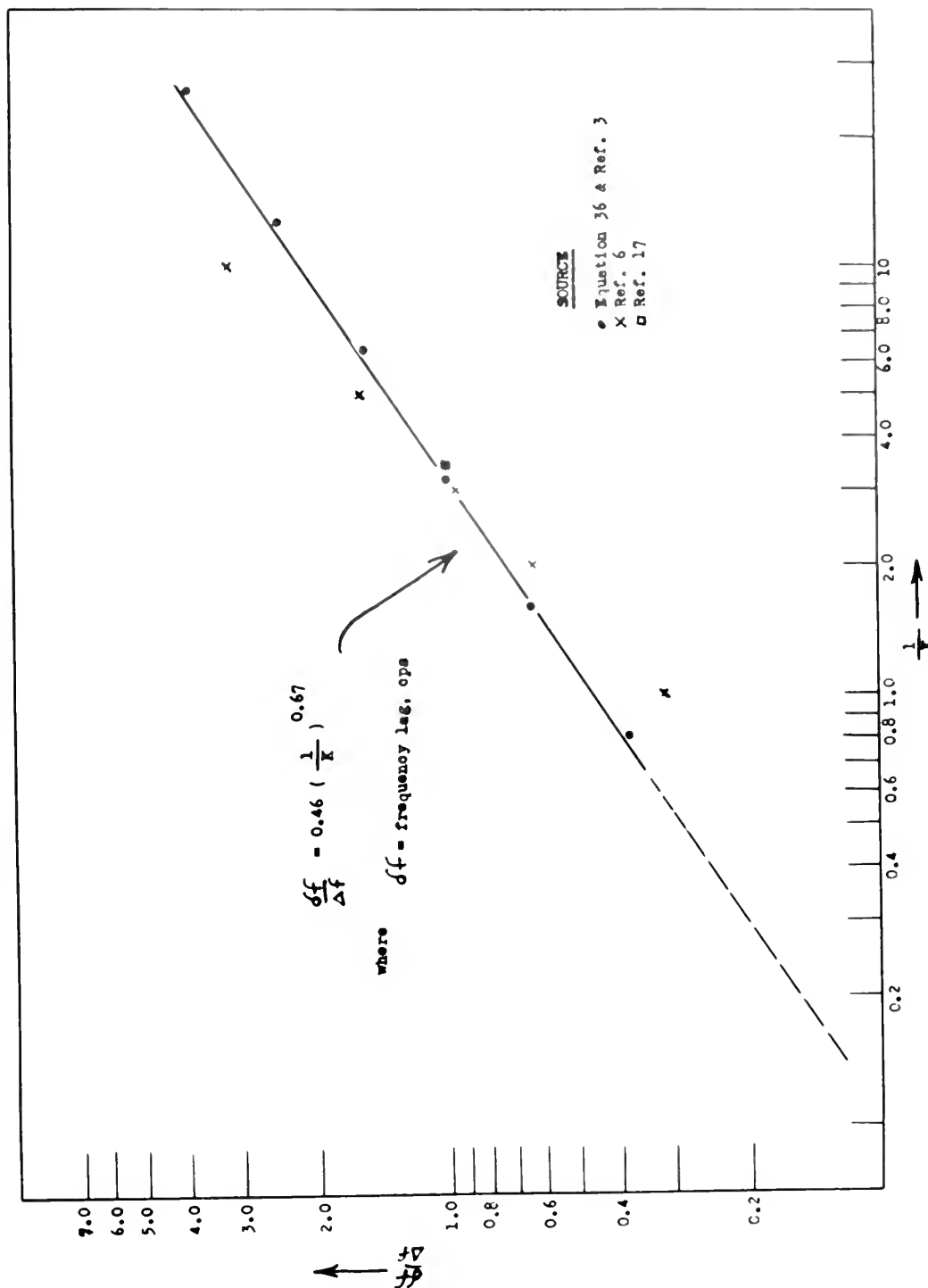






FIGURE 3.5  
 RELATIONSHIP BETWEEN: EFFECTIVE BAND-WIDTH,  $(\Delta f)_e$ , AND NOMINAL BAND-WIDTH,  $(\Delta f)$ , FOR CONTINUOUS ANALYSIS

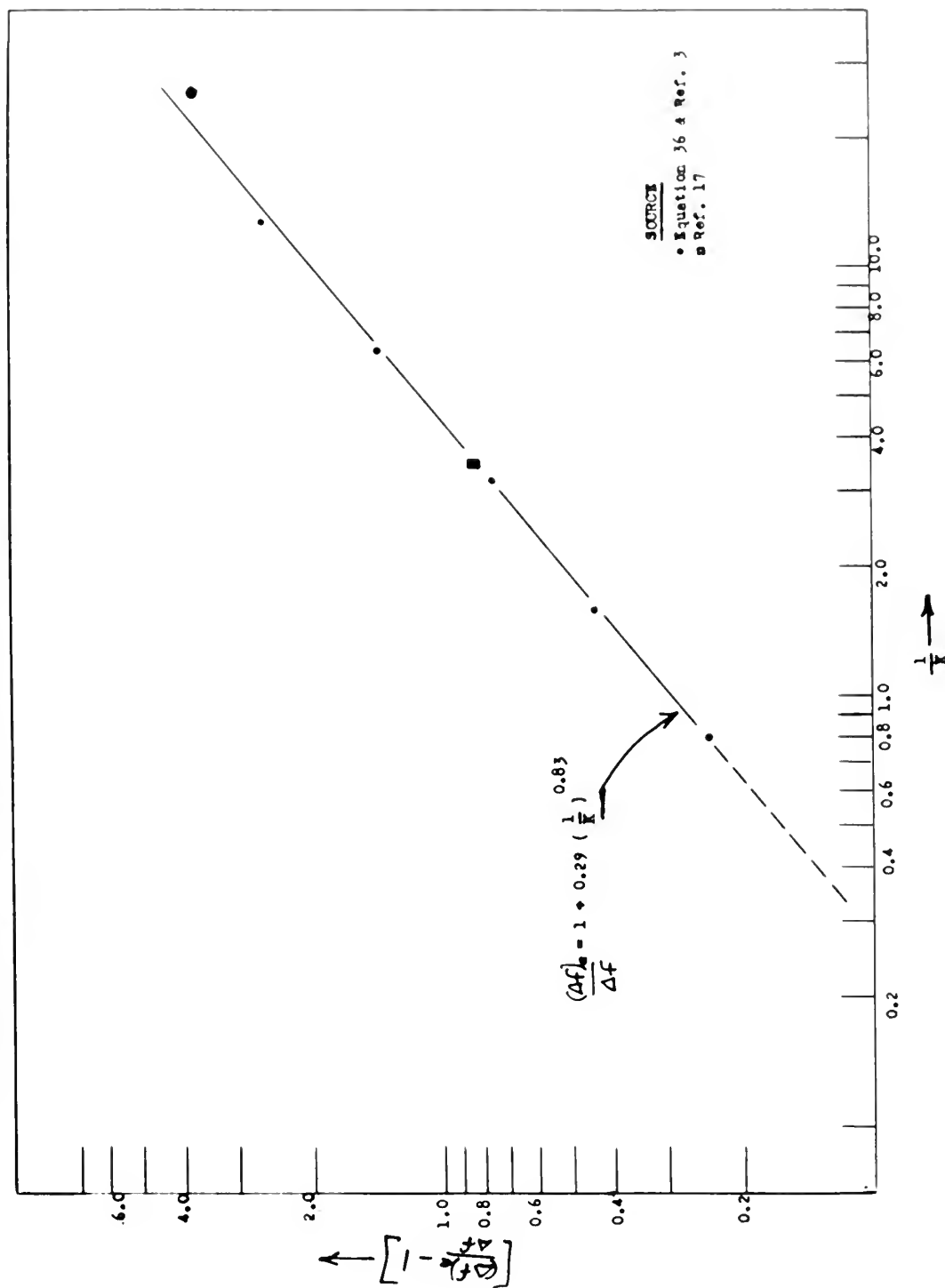




FIGURE 3.6  
Analyzer Indication Of recorded Components

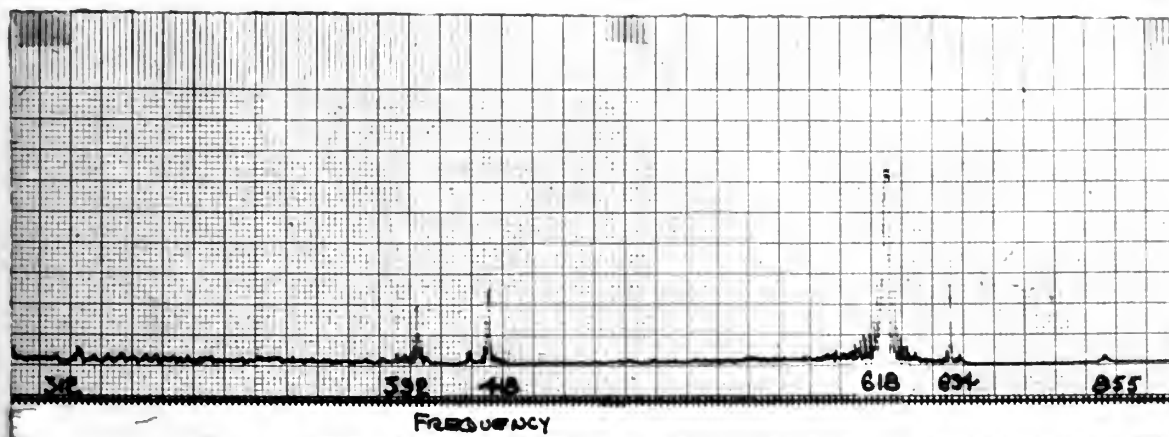
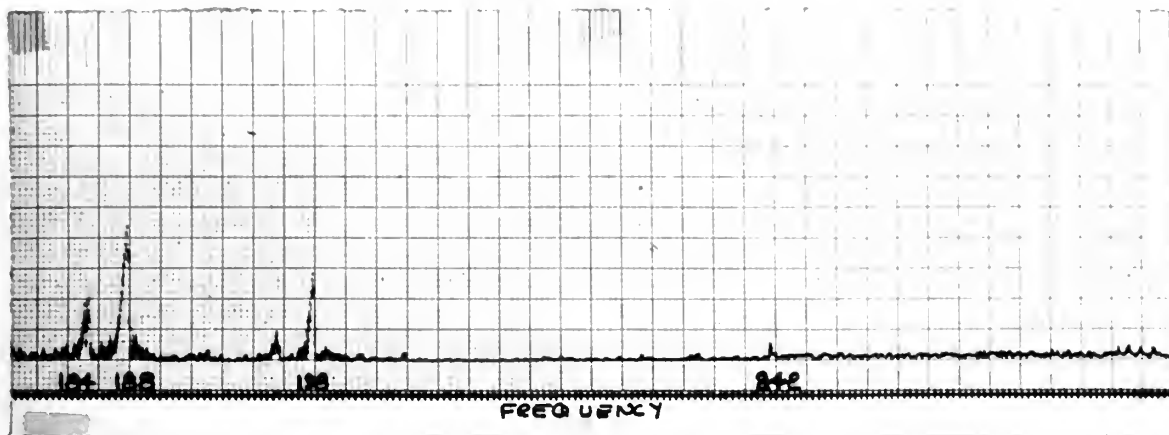




FIGURE 3.7

Expanded Indication of Two Recorded Components  
Showing Resolution of Analyzer

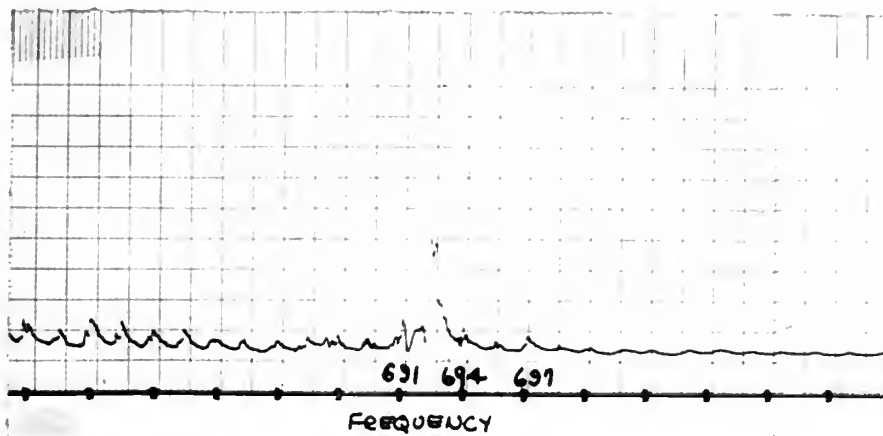
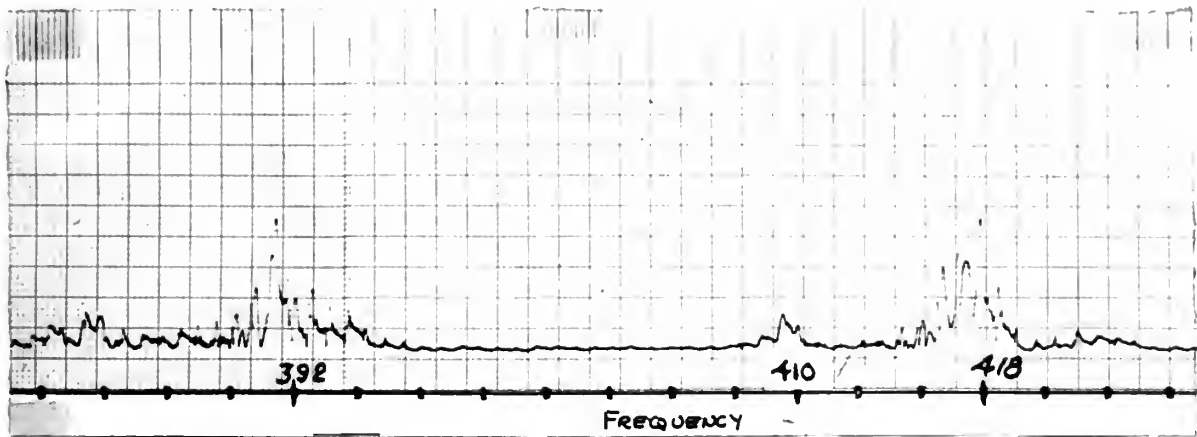
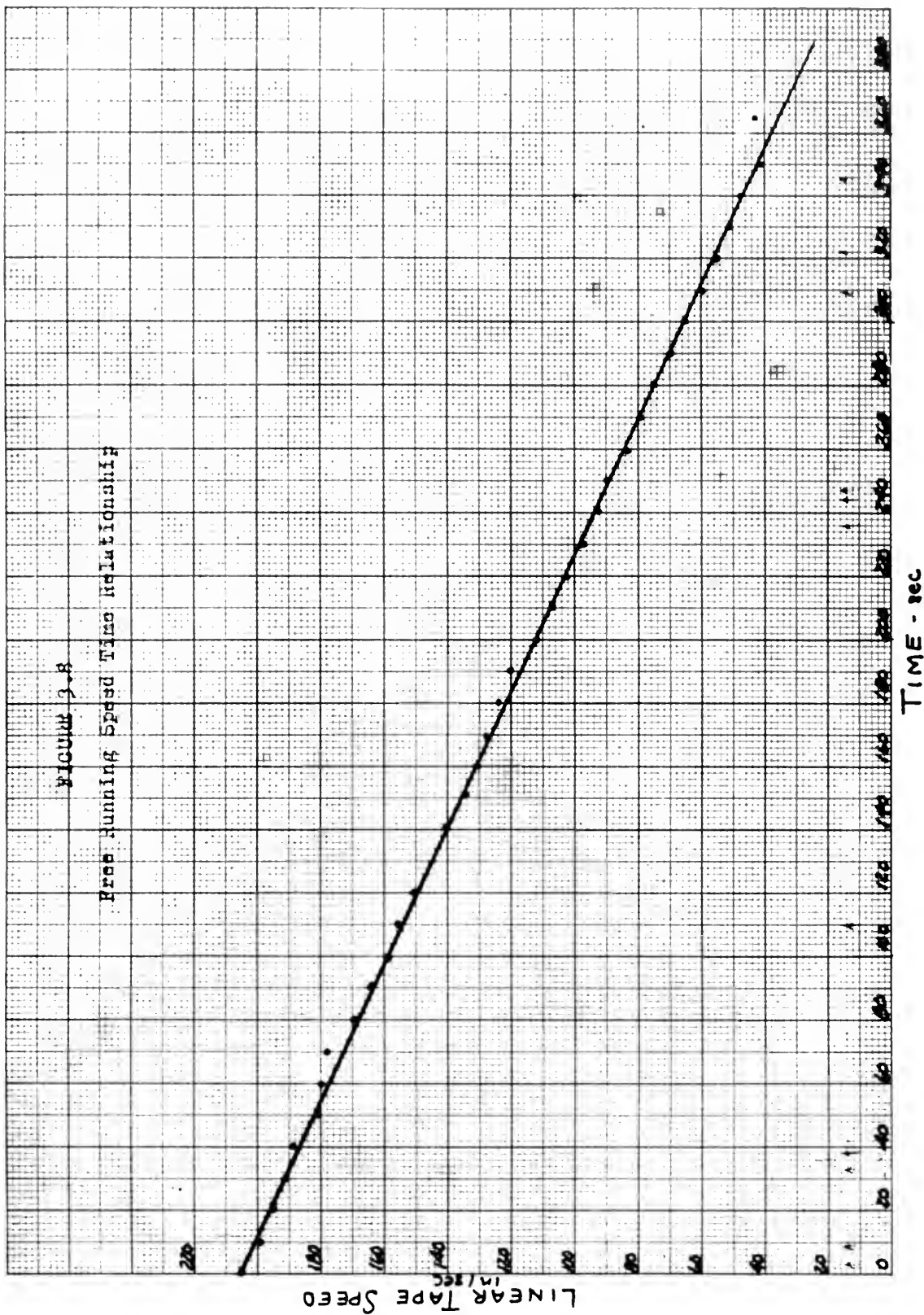


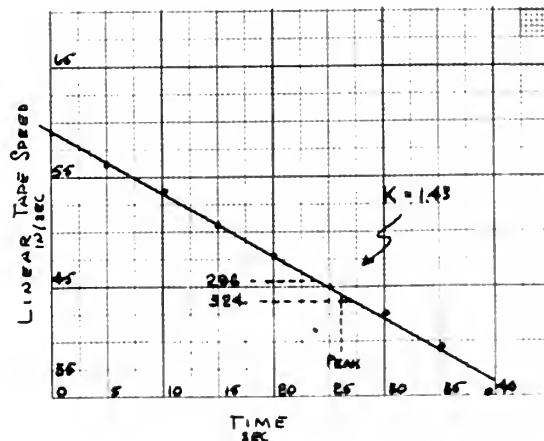
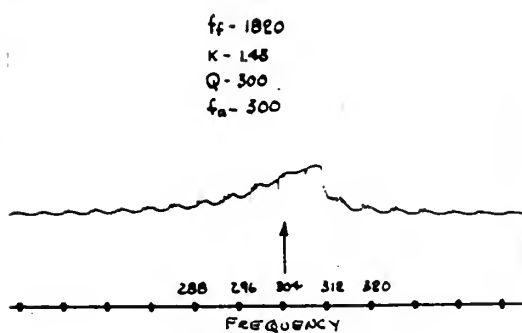
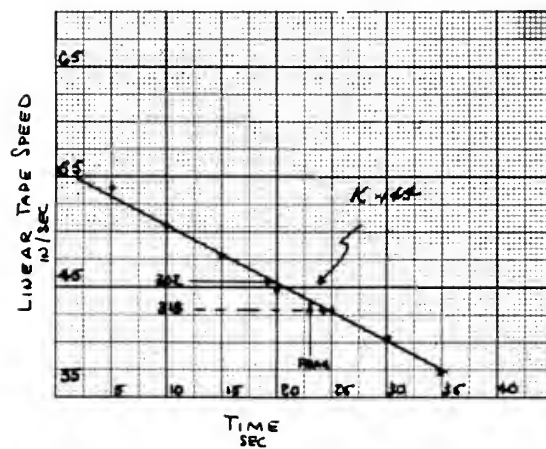
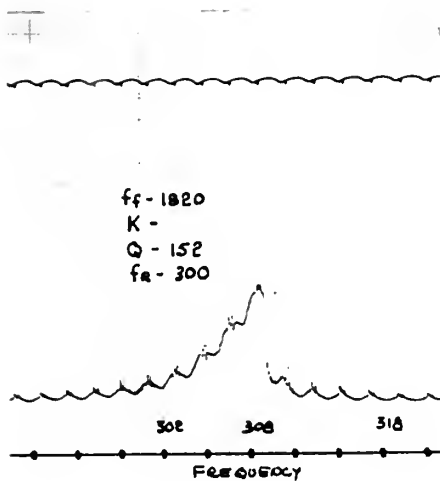
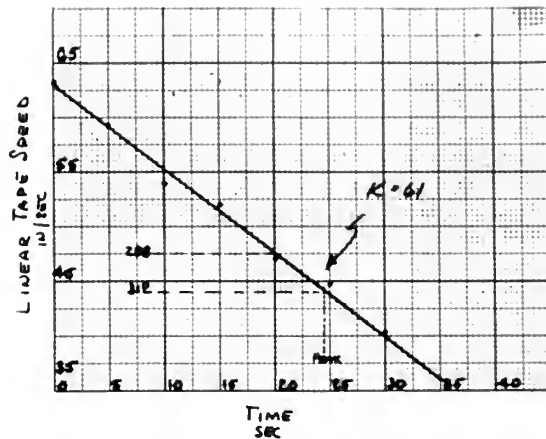
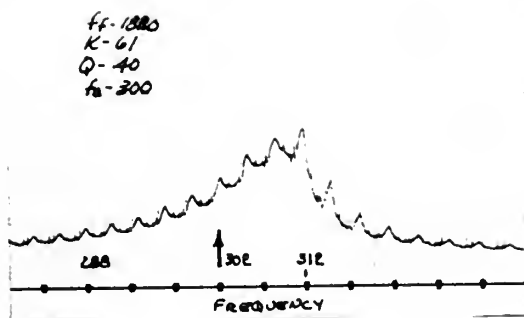


FIGURE 3.8  
Free Running Speed Time Relationship











## CHAPTER IV

### DISCUSSION AND CONCLUSIONS

#### Discussion of Theoretical Results

A basic analyzer theory has been formulated in Chapter II-A, and in Appendices B, C, and D. In general, the Theory provides a basis for answering the questions raised at the end of Chapter I.

#### Which Speed-Time Relationship?

Two basic relationships have been determined: the minimum-analysis-time solution and the equal-sample analysis solution. The equations were derived by a qualitative approach. Fundamentally these are based on the fact that the excitation frequency sweep is very nearly linear while within the pass-band of the high-Q selective network. The deviation from linearity is less than  $1/Q$ .

A basic criterion for any automatic analyzer is to complete the analysis in the minimum possible time compatible with the desired analyzer characteristics. This basic need prompted an investigation into the response of a simple selective network to a nearly linear sweep excitation function. Figures 3.3 to 3.5 correlate the results of Appendix B with other theoretical studies currently published in the literature. Note that these curves are only good for the range of  $K$  indicated. For this analyzer a certain minimum value of  $K$  is considered essential in order that secondary masking does not predominate. Barber<sup>3</sup> concentrates on wider ranges in  $K$  and presents correction values for the various distortions up to  $K$ 's less than  $1/25$ . Our plots are presented in order to provide a basis for establishing tolerable limits of distortion. It is recommended that  $K$  be equal to or greater than 1.0. This should result in peak power errors less than 1.0 db for most selective

1. The first step is to determine the

2. The second step is to determine the  
3. The third step is to determine the  
4. The fourth step is to determine the  
5. The fifth step is to determine the

Which Group-Theory Relationship?

Two basic relationships have been determined: the minimum-relationship  
time solution and the equal-sample analysis solution. The equations were  
derived by a qualitative approach. Fundamentally these are based on the  
fact that the oscillation frequency away is very nearly linear while within  
the pass-band of the high-Q selective network. The deviation from linearity  
is less than 1/10.

A basic criterion for any automatic analyzer is to complete the analysis  
in the minimum possible time compatible with the desired analyzer character-  
istics. This basic need prompted an investigation into the response of a  
simple selective network to a nearly linear sweep excitation function.  
Figures 1.1 to 1.5 compare the results of Appendix 3 with other theoretical  
studies currently published in the literature. While these curves are  
only good for the range of  $\omega$  indicated, for this analyzer a certain minimum  
value of  $\omega$  is considered essential in order that secondary oscillations do not  
predominate. In many cases  $\omega$  is in the range of 10 and presents considerable  
values for the various distortions up to 1/100. For these and  
presented in order to provide a basis for evaluation of analyzer limits of  
distortion. It is recommended that  $\omega$  be equal to or greater than 100. This  
should result in good power output (about 1.0 to 1.5) for each selective

networks which would be used in an analyzer of this type.

Significantly, one of the investigators<sup>17</sup> investigated the response of the network indicated in Figure B.3 to a logarithmic frequency excitation. (All other references are concerned with linear-frequency-sweep excitation.) Penfield's circuit only had a Q of 5, for which there exists a maximum deviation from linearity of 20%. Nevertheless, band-width and frequency lag distortions are exactly predictable by the equations indicated in Figures 3.4 and 3.5, while the peak power error is only 0.2 db from its predicted value. (See Figure 3.3)

Which multiplier relationship should be used in the designed analyzer? Equal sample analysis is a very desirable characteristic. This would avoid the possible time-distribution ambiguity in measurements discussed in Appendix C. Equal sample analysis is considered to be an essential characteristic for the proposed analyzer. Currently available wave analyzers do not achieve it and this would be a big selling-point in favor of this constant-percentage resolution device. Yet, it should be noted from Figures 3.1 and 3.2 that equal sample analysis sometimes requires excessive analysis time as compared with the minimum-analysis-time solution. Another solution would be to increase the number of frequency bands analyzed. However, increased numbers of filters and recorder channels tends to over-complicate what is basically a very simple device. It is anticipated that the wave analyzer could quite possibly consist of a multiplier speed-time relationship made up in part by segments based on both equal-sample-analysis and minimum-analysis-time requirements.

Technically the equal-sample-analysis solution does not provide constant percentage resolution since K does not stay constant. Figure 3.5 indicates that effective band-width becomes smaller, and percentage resolution improves as K increases. Yet for all practical purposes



constant-percentage resolution results. For a high-Q network the difference in resolution for a frequency band is small.

### The Influence of $N(t)$

The speed-time relationship directly affects the averaging and indicating devices. This is clearly indicated in Figure 1.1.

Consider the indicator. The graphic recorder must know what frequency is being analysed at any instant. It is desirable that; (1) the recorder tape have equal intervals of frequency divisions, and (2) this tape be a pre-printed affair. For the minimum-analysis-time solution the recorder paper can move at a constant speed. An amplitude versus  $\log f$  plot results. However, for the desired equal-sample-analysis solution the paper will not run at a constant speed, but instead will follow its own speed-time relationship.

This complication extends to the averaging device. For example in equal sample analysis the 600 cps component remains within the averaging device three times as long as the 200 cps component. The necessary design requirements for the averaging device must be investigated, as must the basic philosophy underlying the averaging circuits for this analyzer.

### Discussion of Experimental Results

The tape for the one run reported for different frequency components has not been analyzed as yet by independent means. It is obvious that there is more on the tape than was supposedly recorded. An oscilloscope was set up to monitor the output of the selective circuit before detection in order to insure that these extra signals were actually being imposed on the selective system. This presentation corroborated the paper recording. The presence of these signals can be explained by in-operative erasing during the recording of the signals. The tape for





14

this analysis was prepared by simultaneous recording of the outputs of several audio oscillators and only the outputs were measured, not the components as actually recorded on the tape. The important thing for this sample is not that extra signals are on the tape but the closeness, with which the several frequencies as recorded were actually analyzed.

During the progress of this thesis it was decided that an amplitude comparison could not be made for the various analyzed frequencies due to (1) the pulse variation due to the eccentricity of the shaft whereby the null of the pulse might add or subtract to the maximum response of a constant amplitude signal, and (2) insertion loss due to the air gap. Nevertheless it is interesting to note the rough correlation between the relative magnitudes of the recording oscillators and those frequency components which did show up on the tape.

A  $Q$  of 625 would give a bandwidth of about 0.3 cycles at 400 cycles. Clearly the resolution which was obtained is not this good. It is an important fact that the  $Q$  of the circuit based only on the half power points is a poor indication of the response of the system. Another important feature of a resonant circuit is the nature of the resonant curve outside the bandpass. These skirts could easily lead to the poorer resolution as obtained with this analyzer.

Actually such a high value of  $Q$  would not be necessary for wave analysis since a resolution of 1% is normally acceptable. The  $Q$  in this case was set high because it was desired to have a  $K$  parameter value of about 1. With the free running case it was necessary to set the  $Q$  up. Due to the difficulty in reading the General Radio frequency meter dial at the resonant frequency of the tuned circuit, it is estimated that this  $Q$  could be in error approaching 100%. However, this

During the process of this study it was noticed that an analysis of the results of the various analyzed frequencies due to (1) the value variation due to the sensitivity of the shift of the null of the phase which aid in adjustment to the maximum response of a constant amplitude of the, and (2) the variation due to the air gap. Nevertheless it is interesting to note the rough correlation between the relative magnitudes of the sensitivity coefficient and those frequency components which show up on the page.

A 2 of 500 would give a bandwidth of about 0.3 cycles at 500 cycles. Clearly the resolution that was obtained is not quite good. It is an important fact that the 2 of the electric band only on the half power points is a poor indicator of the response of the system. Another important feature of a resonant circuit is the nature of the resonant curve outside the resonance. These characteristics really lead to the poorer resolution in comparison with this analysis.

Actually even a 100 value of 2 would not be necessary for some analysis since a resolution of 10 is normally acceptable. The 2 in this case was not high because it was needed to have a 2 parameter value of about 1. The two frequency case is not necessary for the 2 of the electric band in finding the resonant band frequency. The relative frequency of the 2 and 100, it is not

value of  $Q$  would not explain the poorer resolution of the analyzer as would be expected by a resonant system specified only by its  $Q$ . From the expanded indication of Figure 3.7 which was obtained on the fast speed of the recorder, it can be seen that another frequency component separated by two cycles would be resolved. This would correspond to a resolution of approximately 0.3%. The absence of the frequency component at 686 cycles per second can only be explained by the fact that the component was not actually recorded and not that it was obscured by the frequency at 696 cycles.

The higher frequencies which were supposedly recorded on the tape would not show up to any large extent. This fact is due to the rapid change of disc speed at the analysis of these higher frequencies (or low disc speed). This fact is not immediately obvious from the speed time relationship of Figure 3.8 but could be seen by the rapid variation of the butt joint markers in this range. Consequently the amplitude of the reproduced signal would be attenuated due to (1) a parameter value of  $K$  less than 1.00, (2) an increase in the insertion loss at higher recorded frequencies and (3) the normal low frequency drop off of a recorded signal when the recorded wavelength approaches and exceeds the dimensions of the reproduce head. Without belaboring these results any more, it is felt that an accurate analysis using this device could be made from the view point of frequency alone and under certain conditions the amplitude of various frequency components could be measured within tolerable limits.

### Conclusions

The experimental investigation of the analysis brought out several real and important drawbacks in the equipment which would be necessary for this analysis. The first is that it is necessary to maintain con-

[illegible]

The highest frequencies which were experimentally recorded on the tape would not show up to any large extent. This then is due to the rapid change of disc speed at the analysis of these higher frequencies (as far as disc speed). This fact is not immediately obvious from the speed-time relationship of figure 3 but would be seen by the rapid variation of the peak height curves in this range. Consequently the magnitude of the frequencies which would be observed due to the constant value of  $\lambda$  less than 1.0, (2) as because in the variation from 0.5 to 1.0 the frequencies are (1) the natural low frequency band of a recorded signal when the recorded frequency approaches and exceeds the diameter of the recorded band. Without belaboring these details any more, it is left that an accurate picture of the tape is given by the view point of the tape and which is a constant value of the frequency of the recorded signal.

The following information is being furnished to you for your information and for your use in the event of an emergency. It is not to be used for any other purpose.

tact between the reproduce head and the tape and at the very least, any air gap should remain constant for the entire length of sample which is being analyzed. Any air gap is accompanied with a considerable equalization problem if it is desired to analyze any moderate range of frequencies. The second essential disadvantage of the analyzer is the necessity to achieve accurate speed control of the disc or tape. With these inherent drawbacks in mind a reassessment of the entire wave analysis problem was made in order to determine if some concrete conclusions might be made. In particular the features of this analyzer were compared to a conventional heterodyne type analyzer for the desirable characteristics of an analyzer. The results of this reassessment are given below:

Characteristic	Heterodyne	Variable Speed Tape
Maximum resolution	Easily accomplished	Easily accomplished
Constant percentage resolution	Would require some device which would vary the Q of the resonant system as the resonant frequency were varied	Inherent
Automatic operation	Can be accomplished	Inherent
Minimum analysis time with automatic operation	Would require multiplication of frequency of samples to take advantage of faster analysis possible at higher resonant frequency	Inherent in that multiplication of frequencies is inherent
Ambiguity problem	Easily accomplished by magnetic tape storage	Can be made inherent

From these results it is seen that the variable speed magnetic tape analyzer inherently embodies many of the desired characteristics. However, not all these characteristics are desired at the same time and the results must be considered from the viewpoint of one or two specific characteristics

The first of these characteristics is the necessity for a certain degree of generality in the analysis. This is because the analysis is not intended to be a detailed study of a particular case, but rather a general study of a class of cases. The second characteristic is the necessity for a certain degree of generality in the analysis. This is because the analysis is not intended to be a detailed study of a particular case, but rather a general study of a class of cases. The third characteristic is the necessity for a certain degree of generality in the analysis. This is because the analysis is not intended to be a detailed study of a particular case, but rather a general study of a class of cases.

Characteristic	Requirement	Variable Speed Type
Maximum resolution	Highly specialized	Highly specialized
Constant resolution	Highly specialized	Highly specialized
Automatic operation	Highly specialized	Highly specialized
Minimum resolution	Highly specialized	Highly specialized
Highly specialized	Highly specialized	Highly specialized

From these results it is seen that the analysis is not intended to be a detailed study of a particular case, but rather a general study of a class of cases. The analysis is not intended to be a detailed study of a particular case, but rather a general study of a class of cases. The analysis is not intended to be a detailed study of a particular case, but rather a general study of a class of cases.

for some applications. Nevertheless, the feature of using magnetic tape and multiplying frequencies in itself makes many of these other characteristics easily obtainable. A corresponding characteristic for the heterodyne would require a much more elaborate device than is used now. Consequently, it is concluded that any more elaborate equipment could just as well be put into making the variable speed magnetic tape work properly. In truth the equipment for the variable magnetic speed analyzer need not be more elaborate than that which was used in this investigation; it need only be more accurate or function more accurately.

A summary of conclusions is given below:

- (1) Frequency measurements with good accuracy and with good resolution are possible provided that the frequency components of the sample to be analyzed are of the same order of magnitude.
- (2) Measurement of the amplitudes of various frequency components accurately can be made provided the problems of air gap variation and tape speed variation can be solved by more precise experimentation or by different equipment.
- (3) Equal sample time analysis would require a detailed investigation of possible averaging devices and indicators to display the information.
- (4) The butt joint is not a factor in the analyzer provided that there is a sufficient length of sample.

#### Recommendations

- (1) Further study should be made of what types of recorded samples this type of device can measure. Analysis was made on the basis of sinusoidal inputs.

...the results of the analysis of the samples of the material under investigation. It is necessary to take into account the fact that the results of the analysis of the samples of the material under investigation may be influenced by the method of sampling, the method of preparation of the samples, the method of analysis, etc. Therefore, it is necessary to take into account the results of the analysis of the samples of the material under investigation only after a thorough analysis of the results of the analysis of the samples of the material under investigation.

A summary of the results of the analysis of the samples of the material under investigation is given below:

- (1) The results of the analysis of the samples of the material under investigation with good accuracy and with good reliability are possible provided that the frequency of the analysis of the samples is as high as possible and the size of the samples is as small as possible.
- (2) The results of the analysis of the samples of the material under investigation with good accuracy and with good reliability are possible provided that the frequency of the analysis of the samples is as high as possible and the size of the samples is as small as possible.
- (3) The results of the analysis of the samples of the material under investigation with good accuracy and with good reliability are possible provided that the frequency of the analysis of the samples is as high as possible and the size of the samples is as small as possible.
- (4) The results of the analysis of the samples of the material under investigation with good accuracy and with good reliability are possible provided that the frequency of the analysis of the samples is as high as possible and the size of the samples is as small as possible.

...the results of the analysis of the samples of the material under investigation.

- (1) The results of the analysis of the samples of the material under investigation with good accuracy and with good reliability are possible provided that the frequency of the analysis of the samples is as high as possible and the size of the samples is as small as possible.
- (2) The results of the analysis of the samples of the material under investigation with good accuracy and with good reliability are possible provided that the frequency of the analysis of the samples is as high as possible and the size of the samples is as small as possible.



- 47
- (2) Investigate the possibility of using a loop of tape which would be transported across a reproduce head rather than affixing the tape to a drum or disc.
  - (3) Further investigation of the possibility of letting the drum or driving mechanism slow down due to its own or artificially introduced damping. This would involve determining an adequate means of presenting of the derived information.
  - (4) Investigation be made of a device which would permit averaging the response of the selective system when equal sample time analysis was being used. An indicator which would present this information should also be investigated.



**APPENDIX**

---

# EXHIBIT

Exhibit A - Sample of a letter from the Department of Justice to the Department of Education regarding the implementation of the Department of Justice's policy on the use of federal funds for the purpose of providing for the education of students with disabilities.

## APPENDIX A

### NOMENCLATURE

c	equals $\frac{(\Delta f)^2}{K f_u}$ ; (sec) <sup>-1</sup>
d	the spacing introduced between reproducing head and magnetic medium; inches.
e(t)	the excitation function of the selective network.
f	the recorded component being analysed at time t; cps
f <sub>a</sub>	recorded frequency components; cps.
f <sub>b</sub>	reproduced frequency components, i.e. the multiplier output; cps
f <sub>f</sub>	the mid-band frequency of the selective network; cps
f <sub>l</sub>	the lower cut-off frequency of the selective network; cps
f <sub>o</sub>	the value of f for t equals zero; cps
f <sub>u</sub>	the upper cut-off frequency of the selective network; cps
h(t)	the unit impulse response of the selective network.
K	equals $\frac{(\Delta f)^2}{df_{lm}/dt}$ ; cycles.
L	length of recorded sample; seconds or cycles
N(.)	the multiplier speed time relationship.
n	takes on values 0, 1, 2, ...
Q	equals $\frac{f_f}{\Delta f}$
r(t)	the response of the selective network.
S(t)	the instantaneous multiplier speed; inches per second.
S <sub>o</sub>	the multiplier speed at t equals zero; inches per second.
S <sub>r</sub>	the constant recording speed; inches per second.
db	decibels.

APPENDIX

THEORY OF THE  
RELATIVE NETWORK

the system is assumed to be linear and time-invariant, and the response is assumed to be steady-state.

the excitation function of the relative network.

the recorded component being analyzed at time  $t$ ; the

recorded frequency components; the

recorded frequency components, i.e., the multiplier output; the

the mid-band frequency of the relative network; the

the lower cut-off frequency of the relative network; the

the value of  $f$  for a given ratio; the

the upper cut-off frequency of the relative network; the

the total relative response of the relative network.

$$f_{\text{mid}} = \frac{f_{\text{upper}} + f_{\text{lower}}}{2}$$

length of recorded sample; number of cycles

the relative network time relationship.

ratio of upper to lower frequencies

$$\frac{f_{\text{upper}}}{f_{\text{lower}}}$$

the response of the relative network.

the recorded component being analyzed; the relative response

the relative response of a given ratio; the relative response

the relative response of a given ratio; the relative response

the relative response

$\Delta f$	the band-width of the selective network; cps. (equals $f_u - f_l$ )
$\Delta t$	the increment of time any component $f_{bn}$ must remain within the pass-band; seconds.
$\beta$	the effective playback gap length for the reproduce head; inches.
$\delta$	the logarithmic decrement.
$\epsilon$	the angular displacement for $t$ equals zero; radians.
$\lambda$	recorded wavelength; inches
$\theta$	angular displacement; radians
$\tau$	the build-up time for an ideal band-pass selective network; seconds.
$\omega$	angular frequency; radians per second.

(1) The first of these is the

fact that the number of people who

are in the position of being able to

do this is increasing.

Another factor which is of importance

is the fact that the number of people

who are in the position of being able to

do this is increasing.

Another factor which is of importance



## Appendix B

### THEORETICAL DETERMINATION OF THE MULTIPLIER SPEED-TIME RELATIONSHIP

The basic system has been described in Chapter II-A. (See Figure 2.1-2.3) It has been assumed that the system input can be represented as a series of  $n$  discrete sinusoidal components, each of frequency  $f_{an}^*$ . The output of the multiplier is then a series of frequency modulated components,  $f_{bn}(t)$ . The multiplier function can be expressed in terms of the instantaneous reproduce speed,  $S(t)$ , and the fixed recording speed,  $S_R$ .

$$N(t) = \frac{S(t)}{S_R} \quad (1)$$

A fundamental multiplier relationship exists:

$$f_{bn}(t)^* = f_{an} N(t) = f_{an} \frac{S(t)}{S_R} \quad (2)$$

This relationship allows us to describe the excitation function of the fixed selective network as follows

$$e(t) = E \cos \theta = E \cos \left( \frac{d\theta}{dt} \right) t = E \cos [2\pi f_{bn}(t)] t \quad (3)$$

where

$E$  = constant magnitude

$\theta$  = angular displacement

### The Minimum-Analysis-Time Solution

The initial step in the logical development of  $N(t)$  is the determination of a minimum-analysis-time solution. This relationship is to be fixed only by the characteristics of the selective network. Each derived component,  $f_{bn}$ , is to remain within the pass-band for the minimum time for acceptable analysis. This condition is based on the requirement that the output of the selective system remain within

---

\* Where subscript  $n$  takes on values 0, 1, 2, ...

1. The first part of the document is a letter from the President of the United States to the Congress, dated January 3, 1862. It is a very long letter, and it contains a great deal of information about the state of the country at that time. It is a very important document, and it is one of the most interesting documents in the collection.

100

... ..

12-1-1965

and to delivery, in addition to the above, the following information will be furnished:

1. The first group of people who are interested in the results of the study are the researchers themselves. They want to know how well the study was conducted and whether the results are reliable and valid. They also want to know how the study can be used to inform future research.

1950年12月15日

1990

THE UNIVERSITY OF CHICAGO

1. The first step in the process is to identify the problem or issue that needs to be addressed. This involves gathering information and understanding the context of the situation.

2. Once the problem is identified, the next step is to define the objectives and goals of the project. This helps to clarify what is to be achieved and provides a clear direction for the team.

3. The third step is to develop a plan or strategy to address the problem. This involves breaking down the problem into smaller, manageable tasks and determining the resources needed to complete each task.

4. The fourth step is to implement the plan. This involves putting the strategy into action and monitoring progress to ensure that the project is on track.

5. The final step is to evaluate the results of the project. This involves assessing the outcomes against the objectives and goals and identifying any lessons learned for future projects.

certain tolerable limits of amplitude and frequency distortion.

Absolute minimum analysis time would result if: (1)  $\frac{df_{bn}}{dt}$  equals a constant within the pass-band for all values of  $f_{bn}(t)$ , and (2) this constant sweep rate is critical for the selective network consistent with tolerable amplitude and frequency distortion.<sup>19</sup>

In the treatment which follows,  $N(t)$  is evaluated by a qualitative approach initially. Finally, the nature and magnitude of the errors involved will be determined.

For an ideal, band-pass selective network, the build-up time,  $\tau$ , is related to the width of the resonant response curve at the cut-off frequencies,  $\Delta f$ , as follows<sup>2</sup>

$$\tau = \frac{1}{\Delta f} \quad (4)$$

Let us assume that the time,  $\Delta t$ , any component  $f_{bn}$  must remain within the pass-band is proportional to the build-up time.

$$\Delta t = K \tau = \frac{K}{\Delta f} \quad (5)$$

where

$K = \text{a pure numeric (cycles)}$

This certainly will provide a specified minimum distortion, providing  $K$  is made large enough to satisfy the critical excitation sweep rate. Whether this relationship will provide the absolute-minimum-analysis-time solution depends upon the sweep linearity within the pass-band.

Divide both sides of Equation 5 by  $\Delta f$  and invert.

$$\text{Then,} \quad \frac{\Delta f}{\Delta t} = \frac{(\Delta f)^2}{K} \quad (6)$$

The fundamental multiplier relationship has been expressed as

$$f_{bn}(t) = f_{an} N(t) = f_{an} \frac{S(t)}{S_R} \quad (2)$$

[illegible][illegible]

(4)  $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED  
DATE 08-01-2001 BY 60322 UCBAW/SJS

(e)

1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26

1990

1. The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1861. It is a formal address, and it is the first of its kind since the signing of the Constitution. The President, James Buchanan, is addressing the Congress, and he is doing so in a very formal and dignified manner. He is discussing the state of the Union, and he is discussing the issues that are facing the country at that time. He is also discussing the role of the President, and he is discussing the responsibilities of the Congress. The letter is a very important document, and it is a very interesting document. It is a document that is worth reading, and it is a document that is worth studying.

1. *Phragmites australis* (Cav.) Trin. ex Steud. *Phragmites australis* (Cav.) Trin. ex Steud.

Differentiate Equation 2, considering  $f_{an}$  constant.

$$\frac{df_{bn}}{dt} = \frac{f_{an}}{S_R} \frac{dS}{dt} \quad (7)$$

For this analyzer

$$fS = f_u S_R = f_o S_o = \text{constant} \quad (8)$$

where

$f$  = the recorded component,  $f_{an}$ , being analyzed  
at time  $t$  (when  $f_{bn}(t) = f_u$ ).

$S$  = the value of  $S(t)$  at time  $t$ .

$f_l$  = the lower cut-off frequency of the selective  
network.

$f_u$  = the upper cut-off frequency of the selective  
network.

$f_o$  = the value of  $f$  at  $t = 0$  (hence,  $f_o = f_{ao}$ ).

$S_o$  = the value of  $S$  at  $t = 0$ .

Therefore, Equation 7 can be expressed as follows

$$\frac{df_{bn}}{dt} = \frac{f_u}{S} \frac{dS}{dt} \quad (9)$$

Let us assume that

$$\frac{\Delta f}{\Delta t} \approx - \frac{df_{bn}}{dt} \quad (10)$$

We can then equate Equations 6 and 9, and integrate

$$\int_0^t dt = \frac{K}{(\Delta f)^2} f_u \int_{S(t)}^{S_o} \frac{dS}{S} \quad (11a)$$

$$t = \frac{Kf_u}{(\Delta f)^2} \ln \frac{S_o}{S} = \frac{1}{c} \ln \frac{S_o}{S}$$

(1)

$$f(x) = \frac{1}{x^2} = x^{-2}$$

Proof

Let  $f(x) = x^{-2}$  and  $g(x) = x^{-2}$  be two functions.

$$f'(x) = -2x^{-3} = -\frac{2}{x^3}$$

$$g'(x) = -2x^{-3} = -\frac{2}{x^3}$$

$f'(x) = g'(x)$  for all  $x \neq 0$ . Therefore,  $f(x) = g(x) + C$  for some constant  $C$ .

Since

$$f(1) = 1^{-2} = 1 \text{ and } g(1) = 1^{-2} = 1$$

we have

$$1 = 1 + C \implies C = 0$$

$$f(x) = g(x) = \frac{1}{x^2}$$

Therefore,  $f(x) = g(x)$  for all  $x \neq 0$ .

$$\frac{1}{x^2} = \frac{1}{x^2}$$

Let  $x = 1$

$$\frac{1}{1^2} = \frac{1}{1^2}$$

As the two sides are equal, we have

$$\frac{1}{x^2} = \frac{1}{x^2}$$

$$\frac{1}{x^2} = \frac{1}{x^2}$$

or,

$$S(t) = S_0 e^{-ct} \quad (11b)$$

where

$$c = \frac{(\Delta f)^2}{Kf_u}$$

Furthermore, use of Equation 8 allows this relationship to be expressed as

$$t = \frac{Kf_u}{(\Delta f)^2} \ln \frac{f}{f_0} = \frac{1}{c} \ln \frac{f}{f_0} \quad (12)$$

Equations 11b and 12 have been non-dimensionalized and plotted in Figure 3.1.

We can now determine the error involved in the approximation of Equation 10. The general expression for  $\frac{df_{bn}}{dt}$  is obtained from Equation 7. Specifically,

$$\frac{df_{bn}}{dt} = - \frac{cf_{bn} S_0 e^{-ct}}{S_R} = - cf_{bn}(t) \quad (13)$$

Let us investigate the specific region of interest:  $f_1 < f_{bn} < f_u$ .

$$\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u} = - cf_u = - \frac{(\Delta f)^2}{K}$$

But, this equals  $-\frac{\Delta f}{\Delta t}$  from Equation 6.

Hence

$$\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u} = - \frac{\Delta f}{\Delta t}$$

It can be seen that the maximum error involved in the original assumption occurs when  $f_{bn}(t) = f_1$ . From Equation (13)

$$\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_1} = - cf_1.$$





The maximum deviation from linearity can be expressed as

$$(\text{Error})_{\text{Max}} = \frac{\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u} - \left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_l}}{\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u}} = \frac{f_u - f_l}{f_u} \quad (14)$$

Since this system will employ a high-Q selective network, the approximation of Equation 10 is a valid one.

Two characteristics of the sweep rate for the minimum-analysis-time solution should be noted. First, when  $f_{bn}(t)$  equals  $f_u$ , the maximum value of  $\frac{df_{bn}}{dt}$  occurs. Furthermore, this maximum value is constant, independent of the value of  $f_{bn}$ . Therefore, if the sweep rate of the derived components is less than a certain critical value when  $f_{bn}(t)$  equals  $f_u$ , tolerable amplitude and frequency distortion will exist throughout the analysis. Secondly,  $\frac{df_{bn}}{dt}$  is approximately equal to  $-\frac{\Delta f}{\Delta t}$  within the pass-band of the selective network. These two quantities are exactly equal at the upper cut-off frequency, and have maximum deviation at the lower cut-off frequency. The maximum deviation from linear frequency sweep is slightly less than  $\frac{1}{Q}$ ,

where

$$Q = \frac{f_f}{f_u - f_l} = \frac{f_f}{\Delta f}$$

$f_f$  = Mid band frequency of selective network

### A Linear Multiplier

To assist in the selection of  $N(t)$ , a linear speed-time relationship will be investigated. Let us continue the intuitive analysis of Chapter II-A. Figures 2.2 and 2.3 represent the general physical relationships involved. These can be replaced by Figures 3.1 and 3.2 which describe a specific linear multiplier. Note that for this case

(14)



time and system will require a high-order network, the system will be unstable if the order is too high.

The representation of the transfer function for the minimum-phase system is given by  $G(s) = \frac{K}{s^n} \prod_{i=1}^m \frac{s + z_i}{s + p_i}$ , where  $K$  is the gain,  $n$  is the order of the system,  $z_i$  are the zeros, and  $p_i$  are the poles. The system is minimum-phase if all poles and zeros are in the left half of the s-plane.

derived response is less than a certain critical value when  $\omega \rightarrow \infty$ . This is the case for a minimum-phase system. The system is minimum-phase if all poles and zeros are in the left half of the s-plane.

the system is minimum-phase if all poles and zeros are in the left half of the s-plane. The system is minimum-phase if all poles and zeros are in the left half of the s-plane.

$$G(s) = \frac{K}{s^n} \prod_{i=1}^m \frac{s + z_i}{s + p_i}$$

where  $K$  is the gain,  $n$  is the order of the system,  $z_i$  are the zeros, and  $p_i$  are the poles.

### A. System Response

is the response of the system to a unit step input. The system is minimum-phase if all poles and zeros are in the left half of the s-plane. The system is minimum-phase if all poles and zeros are in the left half of the s-plane.

FIGURE B.1

A LINEAR SPEED-TIME RELATIONSHIP,  $N(t)$

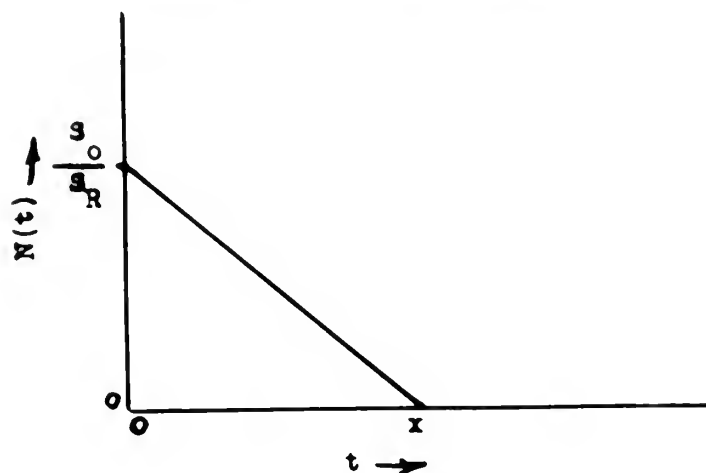
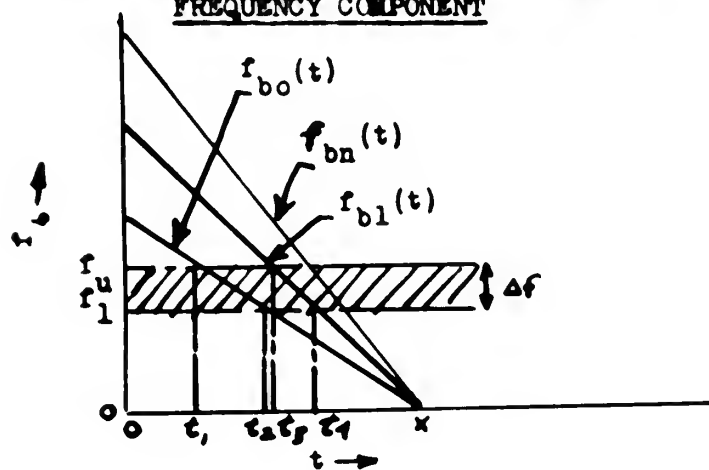


FIGURE B.2

LINEAR FREQUENCY MULTIPLIER OUTPUT  
AS A FUNCTION OF TIME AND RECORDED  
FREQUENCY COMPONENT





$f_{bn}(t)$  is always linear. However, its slope is not of constant magnitude, but is a function of the value of the recorded component being analyzed.

That is,

$$\frac{df_{bo}}{dt} < \frac{df_{bl}}{dt} < \frac{df_{bn}}{dt}_{\max}$$

Although  $\left(\frac{df_{bn}}{dt}\right)_{\max}$  is the critical sweep rate, all other values of  $\frac{df_{bn}}{dt}$  will oversatisfy the characteristic time requirements of the selective network.

We are interested in determining the function  $N(t)$  which will satisfy the following requirements:

- (1) It must be a linear speed-time relationship.
- (2)  $\left(\frac{df_{bn}}{dt}\right)_{\max}$  must equal the critical sweep rate of the selective network.

From Equations 7 and 13,

$$\left(\frac{df_{bn}}{dt}\right)_{\max} = \frac{(f_{an})_{\max}}{S_r} \frac{dS}{dt} = -cf_u$$

Hence,

$$\int_{S_0}^{S(t)} dS = -\frac{cf_u S_r}{(f_{an})_{\max}} \int_0^t dt$$

and,

$$S(t) = S_0 - \frac{cf_u S_r}{(f_{an})_{\max}} t = S_0 \left[ 1 - \frac{cf_0 t}{(f_{an})_{\max}} \right] \quad (15a)$$

where

$$S_0 = \frac{f S_r}{f_0}$$

This can be re-expressed as

$$t = \frac{(f_{an})_{\max}}{cf_0} \left[ 1 - \frac{S(t)}{S_0} \right] \quad (15b)$$

$$\frac{d}{dt} \left( \frac{1}{t} \right) = -\frac{1}{t^2}$$

will have the same derivative as the derivative of the function

as it is stated in the theorem. The function  $f(t)$  will satisfy the following conditions:

- (1) It will be a linear differential equation.
- (2)  $\left( \frac{1}{t} \right)'$  will equal the right-hand side of the equation.

From the above we have

$$\left( \frac{1}{t} \right)' = -\frac{1}{t^2}$$

$$\left( \frac{1}{t} \right)' = -\frac{1}{t^2}$$

and

$$\left( \frac{1}{t} \right)' = -\frac{1}{t^2}$$

where

$$\left( \frac{1}{t} \right)' = -\frac{1}{t^2}$$

Also, from Equation (8),

$$t = \frac{(f_{an})_{max}}{cf_0} \left[ 1 - \frac{f_0}{f} \right]. \quad (15c)$$

A non-dimensionalized plot of Equation 15 is presented in Figure 3.1.

### The Equal-Sample-Analysis Solution

The second step in the formulation of an appropriate multiplier function is to consider those desirable features which might be incorporated in this new analyzer. Previously we have only concerned ourselves with the limitations of the selective network in order to obtain maximum resolution within minimum analysis time. In addition it is desirable that each analyzed component remain under scrutiny for the entire length of sample. This requirement would result in the energy reported at each frequency being associated with the same analysis sample as the energies reported for all other frequencies. Currently available types of wave analyzers do not accomplish this feat. The possible ambiguity in measurements which can result is discussed in Appendix C.

Equal sample analysis can be described as

$$S_0 \Delta t_0 = S \Delta t = \text{constant}. \quad (16)$$

where

$\Delta t_0$  = the time component  $f_{bo}$  must remain  
within the pass-band.

In the minimum-analysis-time solution,  $\Delta t$  is constant; however,  $S$  is continually decreasing. The analysis of the lowest frequency component traverses the entire length of sample. On the other hand, higher frequency analyses cover progressively smaller portions of the original sample. This can lead to an ambiguity in measurements as discussed in Appendix C. A similar appraisal of the linear multiplier relationship

Figure 1.1. A schematic diagram of the experimental setup.

### The Experimental Setup

The second step in the formulation of an appropriate multiplier function is to consider the possible functions which might be incorporated in this new multiplier. Previously we have only concerned ourselves with the limitations of the selective method in order to obtain maximum resolution within minimum analysis time. In addition it is desirable that each analyzed component retains maximum sensitivity for the entire length of analysis. This requirement would result in the energy reported at each frequency being associated with the same analysis sample as the energies reported for all other frequencies. A variety of available types of wave analyzer or wave separator can be used. The available sensitivity in measurement of the wave results is determined by the type of analyzer used. Ideal wave analysis can be achieved by

(16)

$$E_{\text{total}} = E_{\text{signal}} + E_{\text{noise}}$$

where

$$E_{\text{total}} = \text{the total power of the wave}$$

$$E_{\text{signal}} = \text{the power of the signal}$$

In the minimum-sensitivity case, the signal is so small that it is completely obscured by the noise. The analysis of the wave frequency component therefore has an infinite length of time. In the maximum-sensitivity case, the signal is so large that it completely obscures the noise. The analysis of the wave frequency component therefore has a finite length of time. In the intermediate case, the signal is of intermediate size and the analysis of the wave frequency component has a finite length of time. In the maximum-sensitivity case, the signal is so large that it completely obscures the noise. The analysis of the wave frequency component therefore has a finite length of time. In the intermediate case, the signal is of intermediate size and the analysis of the wave frequency component has a finite length of time. In the minimum-sensitivity case, the signal is so small that it is completely obscured by the noise. The analysis of the wave frequency component therefore has an infinite length of time.



indicates that it also presents unequal sample length analyses.

It is possible to evaluate  $N(t)$  in the qualitative fashion which has been followed previously. Equations 5, 8, and 15 can be combined to yield

$$\Delta t = \frac{S_0 \Delta t_0}{S} = \frac{f}{f_0} \Delta t_0 = \frac{K}{\Delta f} \frac{f}{f_0} \quad (17)$$

Divide both sides of the equation by  $\Delta f$  and invert.

$$\frac{\Delta f}{\Delta t} = \frac{(\Delta f)^2 f_0}{K f} = \frac{(\Delta f)^2 S}{K S_0} \quad (18)$$

Let us assume that

$$\frac{\Delta f}{\Delta t} \sim - \frac{df_{bn}}{dt} \quad (19)$$

From Equation (9) and (18),

$$\frac{df_{bn}}{dt} = \frac{f_u}{S} \frac{dS}{dt} = - \frac{(\Delta f)^2 S}{K S_0} \quad (20)$$

Integrating,

$$\int_{S_0}^{S(t)} \frac{1}{S^2} dS = \frac{(\Delta f)^2}{K S_0 f_u} \int_0^t dt$$

or

$$t = \frac{K f_u}{(\Delta f)^2} \left( \frac{S_0}{S} - 1 \right) = \frac{1}{c} \left( \frac{S_0}{S} - 1 \right) \quad (21a)$$

Therefore,

$$S(t) = \frac{S_0}{1+ct} \quad (21b)$$

Similarly,

$$t = \frac{1}{c} \left( \frac{f}{f_0} - 1 \right) \quad (21c)$$

A non-dimensionalized plot of Equation 21 is presented in Figure 3.1.

It is now possible to examine the error resulting from the approximation of Equation 19. Differentiate Equation 21b.

Let  $f(x) = x^2 - 1$  and  $g(x) = x - 1$ .

Then  $f(x) = g(x) \cdot h(x)$  for some polynomial  $h(x)$ .

(17)

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

By the division algorithm, we can write  $f(x) = g(x) \cdot h(x) + r(x)$  where  $\deg(r(x)) < \deg(g(x))$ .

(18)

$$f(x) = (x-1)h(x) + r(x)$$

Let us divide  $f(x)$  by  $g(x)$ .

(19)

$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2-1} \\ \underline{x-1} \phantom{0} \\ 0 \end{array}$$

From the division, we get  $h(x) = x+1$  and  $r(x) = 0$ .

(20)

$$f(x) = (x-1)(x+1) + 0$$

Therefore,

$$f(x) = (x-1)(x+1)$$

or

(21)

$$\left( \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right) = \left( \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right) \left( \begin{matrix} x+1 \\ x-1 \end{matrix} \right)$$

Thus,

(22)

$$(x+1)(x-1) = x^2 - 1$$

which is the required result.

(23)

$$\left( \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right) = \left( \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right) \left( \begin{matrix} x+1 \\ x-1 \end{matrix} \right)$$

A matrix  $A$  is invertible if and only if  $\det(A) \neq 0$ . In this case,  $\det(A) = 1 \cdot 1 - 0 \cdot 0 = 1 \neq 0$ .

3.1.

Let  $f(x) = x^2 - 1$  and  $g(x) = x - 1$ . Then  $f(x) = g(x) \cdot h(x)$  for some polynomial  $h(x)$ .

By the division algorithm, we can write  $f(x) = g(x) \cdot h(x) + r(x)$  where  $\deg(r(x)) < \deg(g(x))$ .

$$\frac{dS(t)}{dt} = - \frac{cS_o}{(1+ct)^2} = - \frac{cS(t)}{1+ct}$$

This expression can be substituted in Equation 7 as follows

$$\frac{df_{bn}}{dt} = - \frac{f_{an}(cS)}{S_r(1+ct)} = - \frac{cf_{bn}(t)}{1+ct} \quad (22)$$

Let us examine the region of interest:  $f_l < f_{bn} < f_u$ .

$$\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u} = - \frac{cf_u}{1+ct} = - \frac{cf_u S}{S_o} = - \frac{(\Delta f)^2 S}{KS_o}$$

But, this equals  $-\frac{\Delta f}{\Delta t}$  from Equation 18.

Hence,

$$\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u} = - \frac{\Delta f}{\Delta t}$$

As in the minimum-analysis-time solution, the maximum deviation from linearity occurs at the lower cut-off frequency of the selective network. From Equation 22

$$\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_l} = - \frac{cf_l}{1+ct}$$

The maximum deviation from linearity is expressed as

$$(\text{Error})_{\max} = \frac{\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u} - \left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_l}}{\left. \frac{df_{bn}}{dt} \right|_{f_{bn}=f_u}} = \frac{f_u - f_l}{f_u} \quad (23)$$

Hence, within the pass-band the maximum deviation from linearity is the same for both the minimum-analysis-time solution and the equal-sample-analysis solution. Its maximum magnitude is less than  $\frac{1}{Q}$ . For all practical purposes, the sweep rate is linear since the analyzer

(10)

Let  $\alpha$  be a root of the equation  $x^2 - 2x + 1 = 0$ .

$$\alpha^2 - 2\alpha + 1 = 0 \implies \alpha^2 = 2\alpha - 1$$

Then, we have  $\alpha^3 = \alpha \cdot \alpha^2 = \alpha(2\alpha - 1) = 2\alpha^2 - \alpha$ .

Since  $\alpha^2 = 2\alpha - 1$ ,

$$\alpha^3 = 2(2\alpha - 1) - \alpha = 4\alpha - 2 - \alpha = 3\alpha - 2$$

As in the previous example, we can find the minimal polynomial of  $\alpha^3$  over  $\mathbb{Q}$ . Let  $\beta = \alpha^3$ . Then  $\beta = 3\alpha - 2$ , so  $\alpha = \frac{\beta + 2}{3}$ .

Substituting  $\alpha = \frac{\beta + 2}{3}$  into the equation  $\alpha^2 - 2\alpha + 1 = 0$ , we get

$$\left(\frac{\beta + 2}{3}\right)^2 - 2\left(\frac{\beta + 2}{3}\right) + 1 = 0$$

The equation simplifies to  $\beta^2 - 4\beta + 4 = 0$ , which is  $(\beta - 2)^2 = 0$ .

$$\beta^2 - 4\beta + 4 = 0 \implies (\beta - 2)^2 = 0$$

$$\beta = 2$$

Therefore, the minimal polynomial of  $\alpha^3$  over  $\mathbb{Q}$  is  $x - 2$ . This means  $\alpha^3 = 2$ .

Since  $\alpha^3 = 2$ , we have  $\alpha = \sqrt[3]{2}$ . The minimal polynomial of  $\alpha$  over  $\mathbb{Q}$  is  $x^3 - 2$ .

system will consist of a high-Q selective network.

Figure 2.3 presents a general picture of the family of decaying components,  $f_{bn}(t)$ , sweeping across a fixed pass-band. The minimum-analysis-time solution exhibits critical sweep rates at  $f_u$  for all values of  $f_{bn}(t)$ . On the other hand, the equal-sample-analysis solution has only one critical sweep rate: this occurs for the derived component  $f_{bo}(t)$  associated with the lowest recorded frequency component,  $f_{ao}$ . All other values of  $f_{bn}(t)$  will oversatisfy the characteristic time requirements of the selective network.

### The Excitation Function, $e(t)$

The excitation function of the selective network,  $e(t)$ , has been described previously as

$$e(t) = E \cos \theta = E \cos (d\theta/dt)t = E \cos [2\pi f_{bn}(t)]t = E \cos [\omega_{bn}(t)]t \quad (3)$$

From equation (2)

$$f_{bn}(t) = f_{an} N(t) = f_{an} \frac{S(t)}{S_r}$$

Table B.1 relates the essential characteristics of  $e(t)$  resulting from the three different multiplier relationships. In order to complete the usefulness of this table, the values of the angular frequency sweep rate  $\frac{d\omega_{bn}}{dt}$  and the ratio  $\frac{df_{bn}/dt}{f_{bn}}$  are provided. Since we are only interested in the critical rate for the linear multiplier, its tabulated characteristics pertain only to  $(f_{an})_{\max} = f_o$ . Note that, wherever possible, reference is made in the table to equations derived in the text.

It will be shown that it is useful to express  $\theta$  by a Maclaurin's Series for the short time interval about  $t = 0$ . Three such series are presented in Table B.2.

1. The first part of the document is a list of names and dates, which appears to be a roster or a list of participants. The names are written in a cursive script, and the dates are written in a more formal, printed style. The list is organized into two columns, with names on the left and dates on the right.

[illegible]

DECLASSIFIED BY: 6032 (2010) DATE: 01/11/2011

... ..

**Abstract**

1955 年 12 月 10 日 星期一 晴

SA 44-1116-1000 1001-7098

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

1970-1971

[illegible]

2011年12月10日 星期一 晴

2019年12月10日

you're looking for a large one in the center of the field, it's a good idea to look for a large one in the center of the field.

*[Faint, illegible handwritten notes]*

[illegible]

...the ... ..

Figure 1

Figure 1. A schematic diagram of the experimental setup. The subject is seated in a chair, viewing a video screen. The screen displays a target (a red dot) and a starting point (a green dot). The subject's hand is positioned at the starting point. The distance between the starting point and the target is 10 cm. The subject is instructed to move their hand from the starting point to the target. The video screen is 100 cm high and 100 cm wide. The starting point is 50 cm from the left edge of the screen. The target is 50 cm from the right edge of the screen. The subject's hand is 50 cm from the left edge of the screen. The distance between the starting point and the target is 10 cm. The subject is instructed to move their hand from the starting point to the target.

1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26

TABLE B.1

## CHARACTERISTICS OF THREE MULTIPLIER SPEED-TIME RELATIONSHIPS

	<u>Minimum-Analysis- Time Solution</u>	<u>Linear * Multiplier</u>	<u>Equal-Sample- Analysis Solution</u>
$S(t)$	$S_0 e^{-ct}$	$S_0 (1-ct)$	$\frac{S_0}{1+ct}$
	[Eq. 11b]	[Eq. 15a]	[Eq. 21b]
$\omega_{bn}(t) = \frac{d\theta}{dt}$	$\omega_{bno} e^{-ct}$	$\omega_{bno} (1-ct)$	$\frac{\omega_{bno}}{1+ct}$
$\theta$	$\frac{\omega_{bno}}{c} (1-e^{-ct}) + \epsilon$	$\frac{-c\omega_{bno} t^2}{2} + \omega_{bno} t + \epsilon$	$\frac{\omega_{bno}}{c} \ln(1+ct) + \epsilon$
$\frac{d\omega_{bn}}{dt}$	$-c\omega_{bno} e^{-ct}$	$-c\omega_{bno}$	$-\frac{c\omega_{bno}}{(1+ct)^2}$
	[Eq. 13]		[Eq. 22]
$\frac{d\omega_{bn}}{\omega_{bn}}$	$-c$	$\frac{-c}{1-ct}$	$\frac{-c}{1+ct}$
where $\omega_{bno} = \frac{2\pi f_{an} s}{s_r}$ ; $c = \frac{(\Delta f)^2}{K_f u}$ ; $\epsilon =$ the value of $\theta$ at $t$ equals 0			

\* with  $(f_{an})_{max} = f_0$

STUDY OF THE EFFECTS OF TEMPERATURE ON THE RATE OF REACTION

REACTANTS  
SODIUM HYDROXIDE

PRODUCTS  
SODIUM CARBONATE

$$(10-1) \cdot 2$$

$$10-1$$

$$(10-1)$$

INITIAL

FINAL

$$(10-1) \cdot 2$$

$$10-1$$

$$(10-1)$$

$$2 + 10-1 = 10-1$$

$$2 + 10-1 = 10-1$$

$$\frac{10-1}{10-1}$$

$$10-1$$

$$10-1$$

$$\frac{10-1}{10-1}$$

$$10-1$$

$$10-1$$

$$10-1 = 10-1$$

$$10-1 = 10-1$$

$$10-1 = 10-1$$



TABLE B.2

 $\theta$  EXPRESSED AS A MACLAURIN'S SERIES

Linear Multiplier:  $\theta = \epsilon + \omega_{bno} t - \frac{\omega_{bno}^2 t^2}{2}$

Minimax-Analysis-Fine Solution:  $\theta = \epsilon + \omega_{bno} t - \frac{\omega_{bno}^2 t^2}{2} + \frac{c^2 \omega_{bno}^3 t^3}{6} - \dots$

Equal-Sample-Analysis Solution:  $\theta = \epsilon + \omega_{bno} t - \frac{\omega_{bno}^2 t^2}{2} + \frac{c^2 \omega_{bno}^3 t^3}{3} - \dots$

TABLE B.3

TRIGONOMETRIC IDENTITIES

1.  $\sin (\alpha-90) = -\cos \alpha$
2.  $\cos (\alpha-\beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
3.  $\cos \alpha \cos \beta = \frac{1}{2} \cos (\alpha-\beta) + \frac{1}{2} \cos (\alpha+\beta)$
4.  $\sin \alpha \cos \beta = \frac{1}{2} \sin (\alpha+\beta) + \frac{1}{2} \sin (\alpha-\beta)$
5.  $\sin (\alpha-\beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$$x^2 - 2x + 1 = 0$$

$$x^2 - 2x + 1 = 0 \quad \text{Factorizing}$$

$$x^2 - 2x + 1 = 0 \quad \text{Factorizing}$$

$$x^2 - 2x + 1 = 0 \quad \text{Factorizing}$$

Answer:

Factorizing

$$1. \quad x^2 - 2x + 1 = 0$$

$$2. \quad x^2 - 2x + 1 = 0$$

$$3. \quad x^2 - 2x + 1 = 0$$

$$4. \quad x^2 - 2x + 1 = 0$$

$$5. \quad x^2 - 2x + 1 = 0$$

## Theoretical Evaluation of the System Response

Reference has been made to a requirement that the output of the selective system remain within certain tolerable amplitude and frequency distortion. From Table B.1 the critical sweep rate for three possible relationships for  $M(t)$  can be expressed as

$$\left( \frac{df_{bn}}{dt} \right)_{\text{critical}} = -cf_u = -\frac{(\Delta f)^2}{K}$$

or

$$K = -\frac{(\Delta f)^2}{\left( \frac{df_{bn}}{dt} \right)_{\text{critical}}} = \frac{(\Delta f)^2}{\left( \frac{df_{bn}}{dt} \right)_{\text{critical}}} \quad (24)$$

It will be shown that the amount of distortion present is a function of the value of  $K$ . In order to evaluate this distortion, it is necessary to determine the system response to an excitation function. This can be accomplished by application of the superposition theorem<sup>7,8</sup> which describes the response of a linear system to an arbitrary excitation function in terms of the response of the system to a unit impulse. This theorem takes the form of the real convolution integral

$$r(t) = \int_{-\infty}^t e(\tau) h(t-\tau) d\tau \quad (25)$$

where

$r(t)$  = the response of the system.

$e(t)$  = the excitation function of the system.

$h(t)$  = the unit impulse response of the linear system.

The analytic treatment of this integral can turn out to be extremely difficult or practically impossible. However, a graphical treatment can be applied in such cases. Gardner and Barnes<sup>7</sup> describe the basic

... ..  
 ... ..  
 ... ..

$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = 1$$

$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = 1$$

... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..

$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = 1$$

... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..  
 ... ..

procedures involved. Such an evaluation is very useful, but it is also very time consuming.

The following analysis is based upon a system which obeys a linear, second order differential equation. To assist in the analytic treatment of Equation 25, the specific LCR circuit of Figure B.3 has been selected. Similar results would be obtained by the use of any linear second order system. The impulse response can be determined from Equation 26 in Figure B.6 by use of Laplace Transform 1.303 from Gardner and Barnes.<sup>7</sup>

$$h(t) = \frac{\omega_0 e^{-\omega_0 t/2Q} \sin \left[ \omega_0 \sqrt{1 - \frac{1}{4Q^2}} t - \psi \right]}{Q \sqrt{1 - \frac{1}{4Q^2}}} \quad (27)$$

or, alternatively

$$h(t) = \frac{\omega_0 e^{-\omega_0 k_1 t}}{Q k_2} \sin (\omega_0 k_2 t - \psi) \quad (27a)$$

where

$$\psi = \tan^{-1} \sqrt{4Q^2 - 1}$$

$$k_1 = \frac{1}{2Q}$$

$$k_2 = \sqrt{1 - k_1^2}$$

In order to evaluate convolution equation 25, it is necessary to describe the excitation function. Table B.2 indicates that for the two non-linear multiplier functions, the first three terms in a Maclaurin's Series of  $\theta$  are identical to a linear frequency sweep. Furthermore, it has been shown that these functions exhibit very little deviation from a linear frequency sweep. (A high-Q selective network exists.) Therefore, the excitation functions can be represented as

$$e(t) \approx E \cos \left( \epsilon + \omega_{bno} t - \omega_{bno} \frac{ct^2}{2} \right) \quad (28)$$

...the ... of ...  
 ...the ... of ...  
 ...the ... of ...  
 ...the ... of ...  
 ...the ... of ...  
 ...the ... of ...

(75) 
$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$$

(76) 
$$\left( \frac{1}{2} + \frac{1}{2} \right) = 1$$

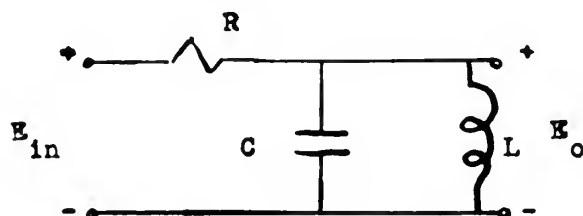
(77) 
$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$$

...the ... of ...  
 ...the ... of ...  
 ...the ... of ...  
 ...the ... of ...  
 ...the ... of ...  
 ...the ... of ...

(78) 
$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$$

Figure B.3

The Selective System Used For Response Evaluation



$$\begin{aligned}
 E_o(s) &= E_{in}(s) \\
 &\quad \frac{1}{1 + RCs + \frac{R}{Ls}} \\
 &= \frac{s E_{in}(s)}{RC \left[ s^2 + \frac{s}{RC} + \frac{1}{LC} \right]}
 \end{aligned}$$

therefore,

$$E_o(s) = \frac{\omega_o s E_{in}(s)}{Q [s + \omega_o / 2Q]^2 + [\omega_o \sqrt{1 - 1/4Q^2}]^2} \dots (26)$$

where

$$\begin{aligned}
 Q &= \frac{R}{L\omega_o} \\
 \omega_o &= \frac{1}{\sqrt{LC}}
 \end{aligned}$$

$E_{in}(s) = 1$  for unit impulse





Substitute  $e(t)$  and  $h(t)$  into Equation 25.

$$r(t) = \frac{\omega_0 E}{k_2 Q} \int_{-\infty}^t \cos \left( \epsilon + \omega_{bno} \tau - \frac{\alpha_1 \omega_{bno}}{2} \tau^2 \right) e^{-\omega_0 k_1 (t-\tau)} \sin [\omega_0 k_2 (t-\tau) - \pi] d\tau \quad (29)$$

For derivation purposes it is not necessary to terminate the series expansion in  $\epsilon$  to a  $t^2$  term as is done in Equation 28. Hok<sup>10</sup> presents a general procedure for solving any transient frequency-modulated signal that can be represented by an excitation vector of the form

$$e = E \exp \{ j \{ \omega_0 t + f(t) \} \}$$

However, this method requires extensive use of the Fresnel integral, which has not been tabulated to any appreciable extent for complex variables.

On the other hand, once Equation 28 is limited to the indicated series expansion, we are confining ourselves to the study of a linear-frequency-sweep excitation. The basic evaluation problem posed by this type of excitation for a simple selective network is not a new one. We attempted several, seemingly-original approaches to the breakdown of Equation 29. However, closer examination of our solutions indicated otherwise. Each approach was merely a variation of one or more procedures contained in recent literature.\* One of these solutions is presented below in order to provide a better understanding of the basic phenomena involved. This approach represents a modification of Barber and Ursell's<sup>2</sup> competent study of a mechanical analogy to our system, which is a variable-speed optical analyzer employing a resonant vibration galvanometer. These authors provide two methods for evaluating Equation 29. The first is exhaustive and determines an upper limit to the errors involved. Although

---

\* These papers are reviewed in Chapter I.



the second method is more superficial, it suggests a physical picture of the response. For this reason the following analysis parallels the latter approach to the problem.

For our wave analyzer Q is very large. For this case

$$\psi \sim 90^\circ$$

$$k_2 \sim 1$$

Therefore, from Trigonometric Identity 1\*

$$h(t) \sim \frac{e^{-\omega_0 k_1 t}}{Q k_2} \cos \omega_0 k_2 t$$

Let

$$\omega_{bno} = \omega_0 = \omega$$

$$A = -E$$

Next, the excitation  $e(t)$ , whose frequency is slowly changing, may be regarded as the sum of two components of constant frequency,  $\frac{\omega}{2\pi}$ , whose amplitudes are slowly changing. From Trigonometric Identity 2, equation 28 can be represented as

$$e(t) = A \cos (\epsilon + \omega t) \cos \left( \frac{\omega t^2}{2} \right) + A \sin (\epsilon + \omega t) \sin \left( \frac{\omega t^2}{2} \right)$$

The effects of these two components will be considered by separate substitution into Convolution Equation 25.

Then

$$r_1(t) = \frac{\omega A}{Q} \int_{-\infty}^t e^{-\omega k_1(t-\tau)} \cos \left( \frac{\omega \tau^2}{2} \right) \cos (\epsilon + \omega \tau) \cos \omega(t-\tau) d\tau$$

Identity 3 allows this to be expressed as

$$r_1(t) = \frac{\omega A}{2Q} \int_{-\infty}^t e^{-\omega k_1(t-\tau)} \cos \left( \frac{\omega \tau^2}{2} \right) [\cos (\epsilon + \omega \tau) + \cos (\epsilon + 2\omega \tau - \omega t)] d\tau \quad (30)$$

Only the second term within the brackets of Equation 30 varies rapidly with

. If we write for this term its mean value of zero,

---

\* All trigonometric identities are listed in Table B.3.

Let  $f: X \rightarrow Y$  be a function.

Suppose  $f$  is injective.

$$f(x) = y$$

Then  $f$  is surjective.  $f(x) = y$  for some  $x \in X$ .

Proof.

$$f(x) = y$$

$$x \in X$$

Let  $y \in Y$ . We want to show that there exists  $x \in X$  such that  $f(x) = y$ . Since  $f$  is injective,  $f$  is also surjective. Therefore, there exists  $x \in X$  such that  $f(x) = y$ .

Q.E.D.

$$f(x) = y \implies x \in X \implies f(x) = y$$

The above proof shows that  $f$  is surjective.  $f(x) = y$  for some  $x \in X$ .

Therefore,  $f$  is bijective.

Q.E.D.

$$f(x) = y \implies x \in X \implies f(x) = y$$

Therefore,  $f$  is bijective.

$$f(x) = y \implies x \in X \implies f(x) = y$$

Therefore,  $f$  is bijective.

$$f(x) = y \implies x \in X \implies f(x) = y$$

Therefore,  $f$  is bijective.

$$f(x) = y \implies x \in X \implies f(x) = y$$

$$r_1(t) = \frac{\omega A}{2Q} \cos(\epsilon + \omega t) e^{-\omega k_1 t} \int_{-\infty}^t e^{\omega k_1 \tau} \cos\left(\frac{\omega \tau^2}{2}\right) d\tau \quad (31)$$

or

$$r_1(t) = \frac{\omega A}{Q} \cos(\epsilon + \omega t) X_1 \quad (31a)$$

Similarly,

$$r_2(t) = \frac{\omega A}{Q} \int_{-\infty}^t e^{-\omega k_1(t-\tau)} \sin\left(\frac{\omega \tau^2}{2}\right) \sin(\epsilon + \omega \tau) \cos \omega(t-\tau) d\tau$$

By Identity 4

$$r_2(t) = \frac{\omega A}{2Q} \int_{-\infty}^t e^{-\omega k_1(t-\tau)} \sin\left(\frac{\omega \tau^2}{2}\right) [\sin(\epsilon + \omega t) + \sin(\epsilon + 2\omega \tau - \omega t)] d\tau \quad (32)$$

Using the same argument as in Equation 31 above,

$$r_2(t) = \frac{\omega A}{2Q} \sin(\epsilon + \omega t) e^{-\omega k_1 t} \int_{-\infty}^t e^{\omega k_1 \tau} \sin\left(\frac{\omega \tau^2}{2}\right) d\tau \quad (33)$$

or

$$r_2(t) = \frac{\omega A}{Q} \sin(\epsilon + \omega t) Y_1 \quad (33a)$$

where  $X_1$  and  $Y_1$  are factors which can be considered as amplitudes which vary slowly with time.

The above expressions can be combined to yield

$$\begin{aligned} r(t) &= r_1(t) + r_2(t) \\ &= \frac{\omega A}{Q} [X_1 \cos(\epsilon + \omega t) + Y_1 \sin(\epsilon + \omega t)] \end{aligned} \quad (34)$$

where

$$\left\{ \begin{array}{l} X_1 \\ Y_1 \end{array} \right\} = \frac{1}{2} \int_{-\infty}^t e^{-\omega k_1(t-\tau)} \left\{ \begin{array}{l} \cos \\ \sin \end{array} \right\} \left( \frac{\omega \tau^2}{2} \right) d\tau \quad (34a)$$

If we change variables so that

$$u = t - \tau$$

then



$$\left. \begin{matrix} X_1 \\ Y_1 \end{matrix} \right\} = \frac{1}{2} \int_0^{\infty} e^{-\frac{\omega u}{2Q}} \begin{matrix} \cos \\ \sin \end{matrix} \left\{ \frac{1}{2} \omega (t - u)^2 du \right. \quad (34b)$$

Equation 34 relates the resonant frequency and the instantaneous frequency of oscillation. It is also convenient to study the excitation frequency,  $\frac{1}{2\pi} \frac{d\theta}{dt}$ . We can define two new variables X and Y where

$$\begin{aligned} X_1 &= - [X \cos \frac{1}{2} \omega t^2 + Y \sin \frac{1}{2} \omega t^2] \\ Y_1 &= - X \sin \frac{1}{2} \omega t^2 + Y \cos \frac{1}{2} \omega t^2 \end{aligned} \quad (35)$$

If these relationships are substituted into Equation 34 we see from Identities 2 and 5 that

$$\begin{aligned} r(t) &= \frac{\omega A}{Q} [Y \sin (\epsilon + \omega t - \frac{\omega t^2}{2}) - X \cos (\epsilon + \omega t - \frac{\omega t^2}{2})] \\ &= \frac{\omega A}{Q} [Y \sin \theta(t) - X \cos \theta(t)] \end{aligned} \quad (36)$$

where

$$\begin{aligned} X &= \frac{1}{2} \int_0^{\infty} e^{-\frac{\omega u}{2Q}} \cos (\omega u - \frac{1}{2} \omega u^2) du \\ Y &= \frac{1}{2} \int_0^{\infty} e^{-\frac{\omega u}{2Q}} \sin (\omega u - \frac{1}{2} \omega u^2) du \end{aligned} \quad (36a)$$

Examination of Equation 36 indicates that the system response is the sum of two oscillations whose frequencies vary like the excitation frequency and whose amplitudes vary slowly with time. In equation 34 we see that

$$\frac{1}{2\pi} \frac{d}{dt} \left[ \tan^{-1} \frac{X_1}{Y_1} \right]$$

is a measure of the difference between the resonant frequency and the instantaneous frequency of oscillation. On the other hand, Equation 36 illustrates that the difference between the excitation frequency and the instantaneous frequency of oscillation can be indicated by

[illegible]



$$\frac{1}{2\pi} \frac{d}{dt} \left[ \tan^{-1} \frac{X}{Y} \right] .$$

Expressions 34 and 36 are equivalent. Both lead to the same value for the amplitude of the response, R.

$$R = [X^2 + Y^2]^{1/2} = [X_1^2 + Y_1^2]^{1/2}$$

With the aid of the Admiralty Computing Service, Barber and Ursell have plotted envelopes of this transient resonance.<sup>2,3</sup> Their results, with appropriate changes in notation, are presented in Chapter II-A.

1. The first part of the report is a general introduction to the subject of the study.

2. The second part of the report is a detailed description of the methods used in the study.

3. The third part of the report is a discussion of the results of the study and their implications.

4. The fourth part of the report is a conclusion and a list of references.

## Appendix C

AMBIGUITY IN WAVE ANALYZER MEASUREMENTS\*

It has, for some time, been recognized that spectrum analyses made by currently available types of wave analyzer apply to wave energy which is varying in character during the analysis. In other words, the energy reported at one frequency is not associated with the same sample of the phenomena analyzed as the energies reported for other frequencies. It is the purpose of this memorandum to examine briefly the nature and magnitude of the errors likely to appear in sound analyses made by instruments of this type.

The oscillograph forming part of Figure C.1 shows, as a function of time, the output of a band-pass filter when responding to the underwater sounds due to the propeller of a passing ship. The output of this filter is restricted to components the frequencies of which lie in the half octave between 212 and 300 cycle/sec. An examination of the outputs of filters passing bands on either side of this indicates that during those time intervals for which a large response is reported the energy spectrum for this band may be virtually continuous and that the energy distribution may be nearly uniform. There is, of course, no assurance of this; a trace having this general appearance would result from a component having a nominal frequency of 250 cycle/sec, but modulated as indicated by the envelope of the trace.

For the frequency range here under consideration it is customary to use an analyzer having a fixed band width of  $\Delta f = 5$  cycles/sec. If this system is to respond properly to any change in energy level it is

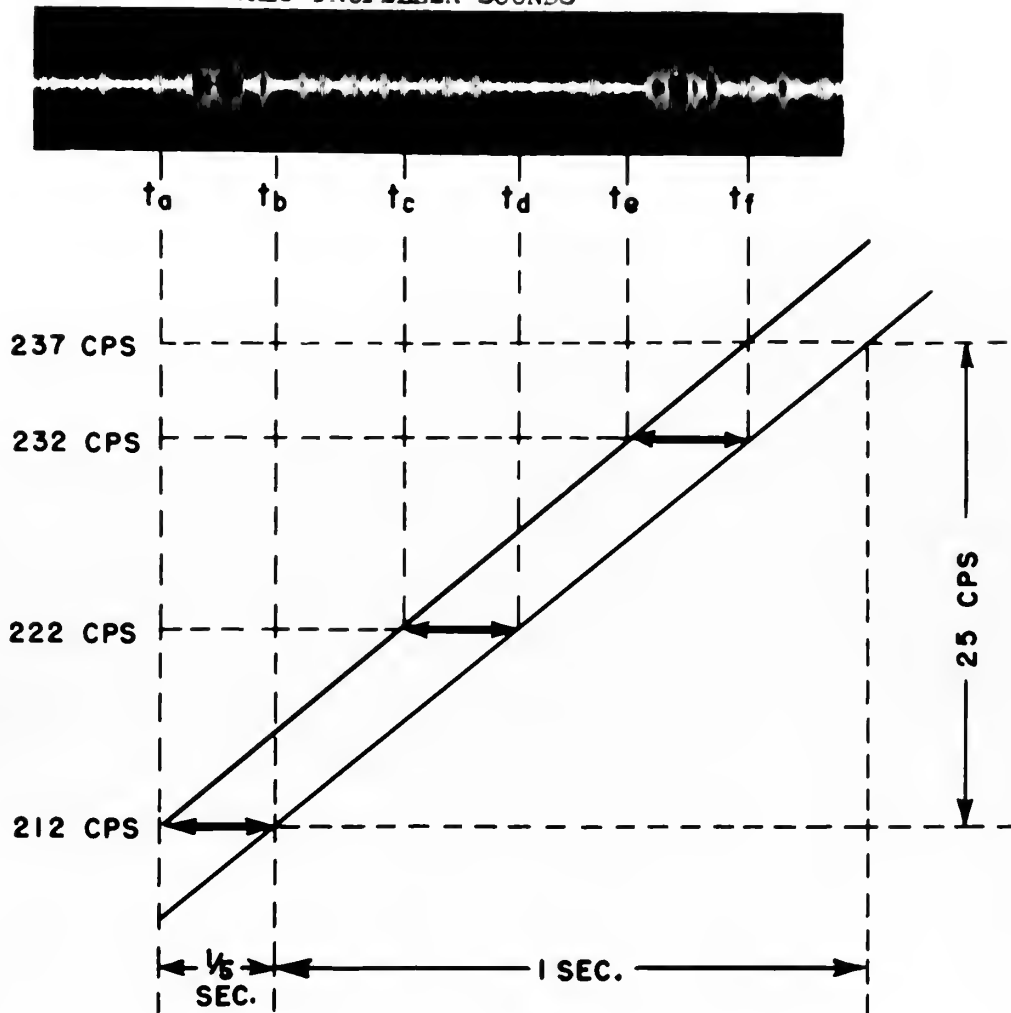
---

\*The discussion which follows is due to Doctor J. W. Horton of the U.S. Navy Underwater Sound Laboratory, Fort Trumbull, New London, Connecticut. (Reference 11.)



FIGURE C.1

RESPONSE OF BAND-PASS FILTER TO UNDERWATER  
SHIP PROPELLER SOUNDS





necessary that each component remain within the pass band for at least  $\Delta t = 1/5$  sec. This fixes the rate at which the spectrum may be scanned at  $\Delta f/\Delta t = 25$  cycle/sec<sup>2</sup>. Now let us assume that the time interval during which the system is responsive to a component having a frequency of 212 cycle/sec is the  $1/5$  sec between the times  $t_a$  and  $t_b$  marked on the oscillogram. The time interval during which it would be responsive to a component having a frequency of 222 cycle/sec would then be the  $1/5$  sec between  $t_c$  and  $t_d$ . If it is indeed a fact that the increase in total energy shown by the oscillograph to occur between  $t_a$  and  $t_b$  results from an increase in the energies of all components within the half octave being analyzed the response of the selective system will show an increase during this interval. This increase will be reported as being associated with a frequency of 212 cycle/sec but not with any particular time. If, during this same interval, the system had been responsive to any other frequency within the half octave band it would have shown a similar increase in response. During the time interval between  $t_c$  and  $t_d$  the selective system will, under the conditions represented by the diagram, show a reduced response. This will be reported as associated with a frequency of 222 cycle/sec. This system would, however, have shown a low response for this time interval if its selective system had been responsive to components at any other portion of the frequency spectrum. In a similar manner the time interval during which the system is responsive to a component having a frequency of 232 cycle/sec is the  $1/5$  sec between  $t_e$  and  $t_f$ . If, again, the increased filter output occurring here represents an increase in the amount of energy associated with all components the analyzer will, once more, show an increased response.

The first part of the paper is devoted to a discussion of the general principles of the theory of the system. It is shown that the system is a linear system and that the response of the system to a step function is a function of the time constant of the system. The second part of the paper is devoted to a discussion of the experimental results. It is shown that the response of the system to a step function is a function of the time constant of the system. The third part of the paper is devoted to a discussion of the theoretical results. It is shown that the response of the system to a step function is a function of the time constant of the system. The fourth part of the paper is devoted to a discussion of the conclusions. It is shown that the response of the system to a step function is a function of the time constant of the system.



Now, however, this will be associated with a frequency of 232 cycle/sec. As the analysis continues, there will be a similar increase in the response; these will be associated with frequencies of 252, 272, and 292 cycles/sec.

The complete analysis, in other words, appears to show that the energy present in the output of the filter is concentrated in five components, spaced by 20 cycle/sec intervals. This may be very far from the truth. In any statement purporting to specify the manner in which energy varies with frequency it is expected that the data associated with one spectrum interval apply to the same sample of the phenomena as do the data associated with other spectrum intervals. In the case here under consideration this would be equivalent to a statement specifying the results of simultaneous observations of the outputs of a series of selective systems, the 5 cycle/sec bands of which cover the entire half octave passed by the original filter. If such observations, or their equivalent, were to be made it might well be found that more energy was associated with a frequency for which the analyzer reported no energy than for a frequency which, by pure chance, happened to be scanned at a time when all components were passing through energy maxima.

In the original oscillogram we have information relative to the time variation in the rate of flow of the total energy for all components within the half octave band. With respect to the energy within successive narrow intervals of the spectrum, however, it is impossible to determine from the data presented by the analyzer to what extent observed changes in energy magnitude are due to changes in frequency or to what extent they are due to changes in time.

In the case postulated above the rate at which pulses of acoustical energy occur may be computed directly from the information obtained

The first of these is the fact that the
 data are not normally distributed. This
 is a problem because many statistical
 tests assume normality. The second
 problem is that the data are
 correlated. This is a problem because
 many statistical tests assume
 independence. The third problem is
 that the data are noisy. This is a
 problem because many statistical tests
 assume that the data are free of
 noise.

by the sound analyzer. Knowing that the frequency spectrum is scanned at a rate of 25 cycles/sec<sup>2</sup> and observing that response maxima occur every 20 cycle/sec, it is evident that there is a maximum every 0.8 sec. There are, in other words, 75 pulses/min. This is in agreement with a count made against the time scale of the oscillogram.

It is interesting to examine the manner in which the situation described above is affected by a change in the resolving power of the analyzer. Suppose, for example, that the band width of the selective system is increased to  $\Delta f = 10$  cycle/sec; the time required for this system to respond to any change in energy level is then  $\Delta t = 1/10$  sec and the spectrum may be scanned at a rate of  $\Delta f/\Delta t = 100$  cycle/sec<sup>2</sup>. Let us assume, as before, that the time interval between two successive energy maxima in the filter output is 0.8 sec. At the increased scanning rate the spectrum interval scanned during this time interval will be 80 cycle/sec. If, therefore, the system is responsive to a frequency of 212 cycle/sec during the time interval coinciding with one energy maximum, it will be responsive to 292 cycle/sec during the time interval coinciding with the next succeeding maximum. The analyzer will now report that the energy output of the filter is concentrated in two components, separated by an 80 cycle/sec spectrum interval. From this it is evident that when the spacing of response maxima is due to a time variation of energy rather than to its frequency distribution, and when the scanning rate is maintained at the maximum value at which the selective system can properly respond to changes in energy level, the length of the spectrum interval reported between response maxima will vary as the square of the band width.

[illegible]

## APPENDIX D

### SAMPLE LENGTH CONSIDERATIONS

The theoretical length of sample required for proper analysis can be evaluated from relationships formulated in Appendix B.

The theoretical minimum length of sample,  $L_{\min}$ , is determined by use of Equation 16.

$$L_{\min} \text{ (seconds)} = \frac{S_o \Delta t_o}{S_r} \quad (37)$$

where

$S_o$  = the multiplier speed at time  $t$  equals 0.

$\Delta t_o$  = the time interval during which component  $f_{bo}$  must remain within the pass-band for proper analysis.

$S_r$  = the constant recording speed.

From Equations 4 and 8 we know that

$$\Delta t_o = \frac{K}{\Delta f} \quad (4)$$

and

$$S_o = \frac{f_u S_r}{f_{ao}} \quad (8)$$

Substitute these quantities into Equation 37. Then

$$L_{\min} \text{ (seconds)} = \frac{K}{f_{ao}} \left( \frac{f_u}{\Delta f} \right) \quad (38)$$

where

$$K = \frac{(\Delta f)^2}{df_{bn}/dt}$$

$f_u$  = the upper cut-off frequency of the selective network.

$f_{ao}$  = the recorded frequency component being analyzed at time  $t$  equals zero.

$\Delta f$  = the band-width of the selective system.

$\frac{f_u}{\Delta f}$  = a measure of the percentage resolution.\*

---

\* For a high-Q system  $\frac{f_u}{\Delta f} = \frac{f}{\Delta f}$ . A 1/2 % error is involved in this approxima-

THEOREM 1.1

Let  $\mathcal{H}$  be a Hilbert space and let  $T$  be a bounded linear operator on  $\mathcal{H}$ . Then the following conditions are equivalent:

(i)  $T$  is self-adjoint, i.e.,  $T = T^*$ .

(ii)  $\langle Tx, y \rangle = \langle x, Ty \rangle$  for all  $x, y \in \mathcal{H}$ .

(iii)  $\langle Tx, x \rangle \in \mathbb{R}$  for all  $x \in \mathcal{H}$ .

(1.1)

$$\langle Tx, x \rangle \in \mathbb{R} \text{ for all } x \in \mathcal{H}.$$

Proof.

(i)  $\Rightarrow$  (ii) Let  $x, y \in \mathcal{H}$ . Then  $\langle Tx, y \rangle = \langle x, Ty \rangle$  because  $T = T^*$ .  
(ii)  $\Rightarrow$  (i) Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .  
(i)  $\Rightarrow$  (iii) Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

(1.2)

$$\langle Tx, x \rangle \in \mathbb{R} \text{ for all } x \in \mathcal{H}.$$

Proof.

(1.3)

$$\langle Tx, x \rangle \in \mathbb{R} \text{ for all } x \in \mathcal{H}.$$

Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

(1.4)

$$\langle Tx, x \rangle \in \mathbb{R} \text{ for all } x \in \mathcal{H}.$$

Proof.

$$\langle Tx, x \rangle \in \mathbb{R} \text{ for all } x \in \mathcal{H}.$$

Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

Let  $x \in \mathcal{H}$ . Then  $\langle Tx, x \rangle = \langle x, Tx \rangle = \overline{\langle Tx, x \rangle}$ , so  $\langle Tx, x \rangle \in \mathbb{R}$ .

Note that the theoretical minimum length of sample can also be expressed as

$$L_{\min}(\text{cycles}) = \frac{K f_u}{\Delta f} \quad (38a)$$

Assume that a one percent resolution is desired. For a 1 - 1000 cycles per second analysis-band,  $100 \times K$  seconds of sample tape is required. However, a 100 - 100,000 cycles per second analysis-band requires only  $1 \times K$  seconds length of tape.

### An Alternative Approach

The direct approach presented below is based on a procedure suggested by Doctor J. W. Horton. The analysis is included since it tends to tie-together the quantities  $\Delta t$  and  $L_{\min}$ .

The logarithmic decrement of a resonant system is defined as

$$\delta = \ln \frac{I_1}{I_2}$$

where  $\frac{I_1}{I_2}$  is the ratio of the amplitude of any two successive cycles:

Yet, on page 139 of Reference 30 we see that

$$\delta = \frac{\pi}{Q}$$

where

$$Q = \frac{f_f}{\Delta f}$$

By combining,

$$\ln \frac{I_1}{I_2} = \frac{\pi \Delta f}{f_f}$$

Hence,

$$\ln \left( \frac{I_1}{I_2} \right)^n = \ln \frac{I_0}{I_n} = \frac{\pi n \Delta f}{f_f}$$

Here  $\frac{I_0}{I_n}$  is the ratio of the amplitude at the beginning of an interval

$\frac{n}{f_f} = \tau$  seconds duration to the amplitude at the end of this interval,

... (a) ... (b) ... (c) ... (d) ... (e) ... (f) ... (g) ... (h) ... (i) ... (j) ... (k) ... (l) ... (m) ... (n) ... (o) ... (p) ... (q) ... (r) ... (s) ... (t) ... (u) ... (v) ... (w) ... (x) ... (y) ... (z) ...

An Alternative Approach

The direct approach presented before is based on a procedure suggested by Doctor J. W. Horton. The analysis is limited since it tends to the logarithm the quantities  $\frac{I_0}{I_1}$  and  $\frac{I_1}{I_2}$ .

The logarithmic derivation of a resonant system is defined as

$$L = \ln \frac{I_0}{I_1}$$

where  $\frac{I_0}{I_1}$  is the ratio of the amplitudes of any two successive cycles:

Yet, on page 119 of Reference 30 we see that

$$\frac{I_0}{I_1} = \frac{1}{Q}$$

where

$$Q = \frac{1}{2\zeta}$$

By combining,

$$\ln \frac{I_0}{I_1} = \ln \frac{1}{2\zeta} = -\ln 2\zeta$$

Hence,

$$\ln \left( \frac{I_0}{I_1} \right) = \ln \left( \frac{1}{2\zeta} \right) = -\ln 2\zeta$$

Here  $\frac{I_0}{I_1}$  is the ratio of two amplitudes at the beginning of an interval

$\frac{1}{2\zeta} = \frac{1}{Q}$  denotes deviation to the amplitude of the undamped system.



Let

$$\frac{n\Delta f}{f} = \tau \Delta f = 1 \text{ cycle}$$

That is, consider an interval proportional to the reciprocal of the bandwidth of the selective system

$$\tau = \frac{1}{\Delta f}^*$$

Then

$$\ln \frac{I_0}{I_n} = \pi$$

$$I_n = 0.0433 I_0$$

Considering the build-up of current in a resonant system, the current would reach 95.7% of its final amplitude during this interval.\*\*

The number of cycles required for this change is given by

$$\ln \frac{I_0}{I_n} = \frac{\pi n}{Q} = \pi$$

Hence

$$n = Q$$

This shows that the time requirement of Equation 4 is equivalent to saying that the sample must contain Q cycles. Equation 38a is equivalent providing K equals 1.

### Butt-weld Considerations

The butt-weld joint influences the sample length as determined in Equation 38. Beranek<sup>4</sup> points out that the butt-weld is usually of insufficient length to effect the analysis providing playback time for the loop is greater than a second or two. We have seen that a certain

---

\* This is Equation 4 of Appendix B.

\*\* An interesting correspondence exists between this result which indicates -0.4 db power error for K = 1, and Figure 3.3 which predicts -0.5 db power error for K = 1.



minimum number of cycles must be considered in order to achieve a proper analysis. The relative location of the joint and the reproduce head for a given analyzed component can prevent achievement of the required  $\Delta t$ . This butt-weld limitation results from a 180 degree phase shift introduced at the joint. Let us arbitrarily increase the sample length by a factor of two; this eliminates the 180 degree phase shift problem. Therefore, let

$$L \text{ (seconds)} = \frac{2 K f_u}{f_{so} \Delta f} \quad (39)$$

$$L \text{ (cycles)} = \frac{2 K f_u}{\Delta f} \quad (39a)$$

A partial method of circumventing errors of this nature is for the analyzer operator to separately investigate two different sample loops. These samples should be prepared with a 90 degrees phase shift relative to the location of the butt weld. If for a particular frequency, one loop presents a broad-band indication and the other loop exhibits a peak indication, the operator is made aware that a butt weld ambiguity exists for the broad-band indication.



## APPENDIX E

### DETAILS OF EXPERIMENTAL EQUIPMENT AND PROCEDURE

#### Introduction

This appendix gives in more detail a description of the experimental equipment. Some of the more pertinent factors in experimental procedure are described. The last part of the appendix discusses other equipment which will accomplish the same functions as the equipment actually used.

#### Disc and Reproduce Head Assembly

The heart of the problem and of the unique feature of derivation of analyzer frequency by multiplication is embodied in this assembly.\* Figure E.1 is a general view of this assembly, showing also the DC driving motor assembly and tachometer. Figure E.2 shows details of the reproduce head mounting. The disc used to transport the tape was 9.00 inches in diameter and was fabricated from brass. The disc had a minimum run-out or eccentricity of .0017 inches as measured by a dial indicator. The minimum used in this connection refers to the fact that various amounts of run-out were possible by varying the relative angular position of the disc and the shaft which turned the disc. Naturally the minimum run-out was used. Three pulleys on the disc shaft and three corresponding pulleys on the driving shaft permitted speed changes of 10:1, 1:1, and 0.5:1 between the driving shaft and the disc.

No little trouble was encountered in attaching the tape to the disc and in setting the face of the reproduce head at the closest possible distance from the magnetic tape. The trouble in connection with attaching the tape stems from the fact that the glue under the tape caused an

---

\* The basic disc and reproduce head assembly were provided by the Recording Branch, U.S. Navy Underwater Sound Laboratory.

10/10/10

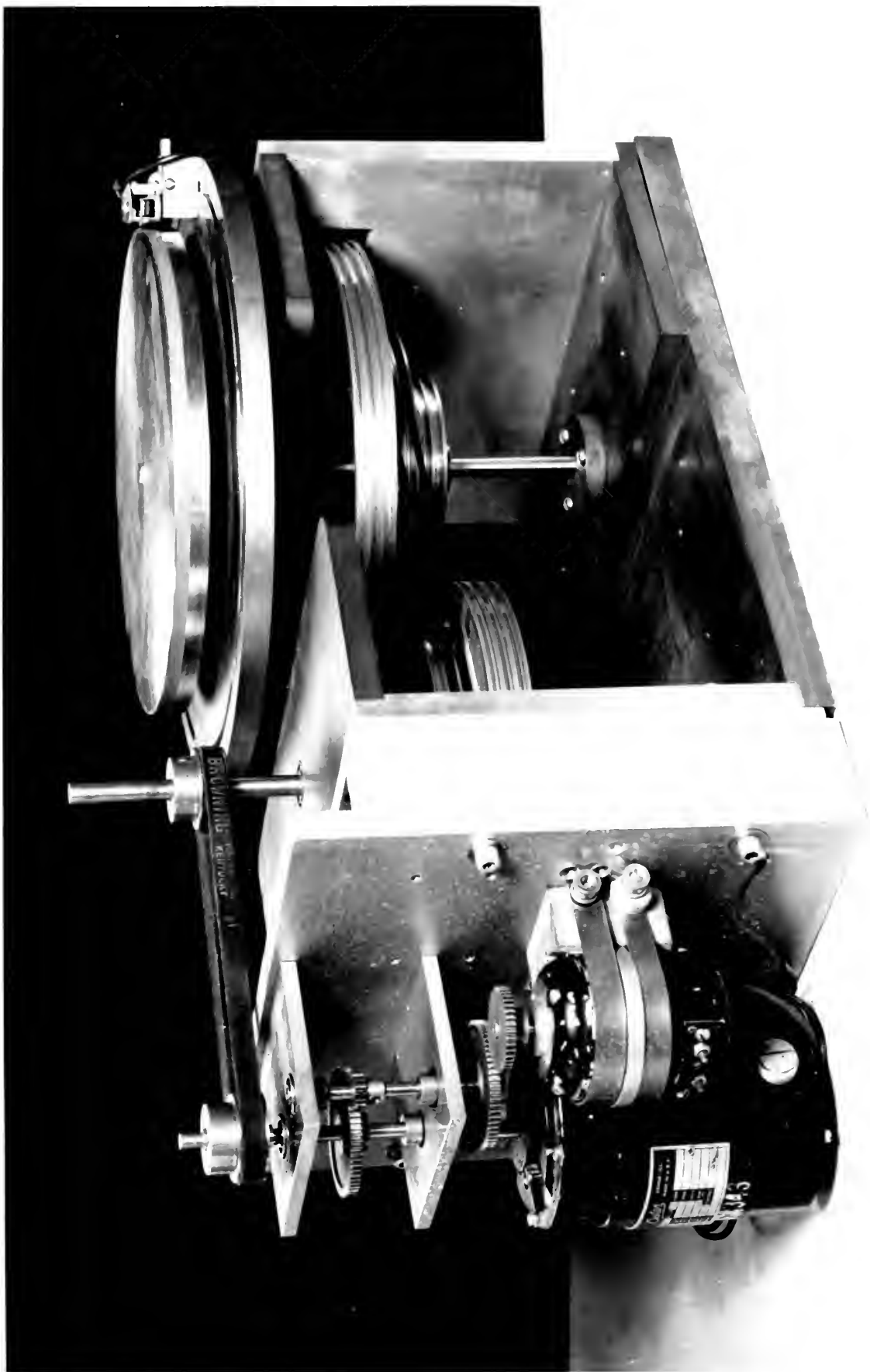
This report is a summary of the results of the investigation of the accident which occurred on 10/10/10. The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report. The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report.

### 1. Introduction

The purpose of the investigation was to determine the causes of the accident and to identify any measures that could be taken to prevent a similar accident from occurring in the future. The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report. The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report.

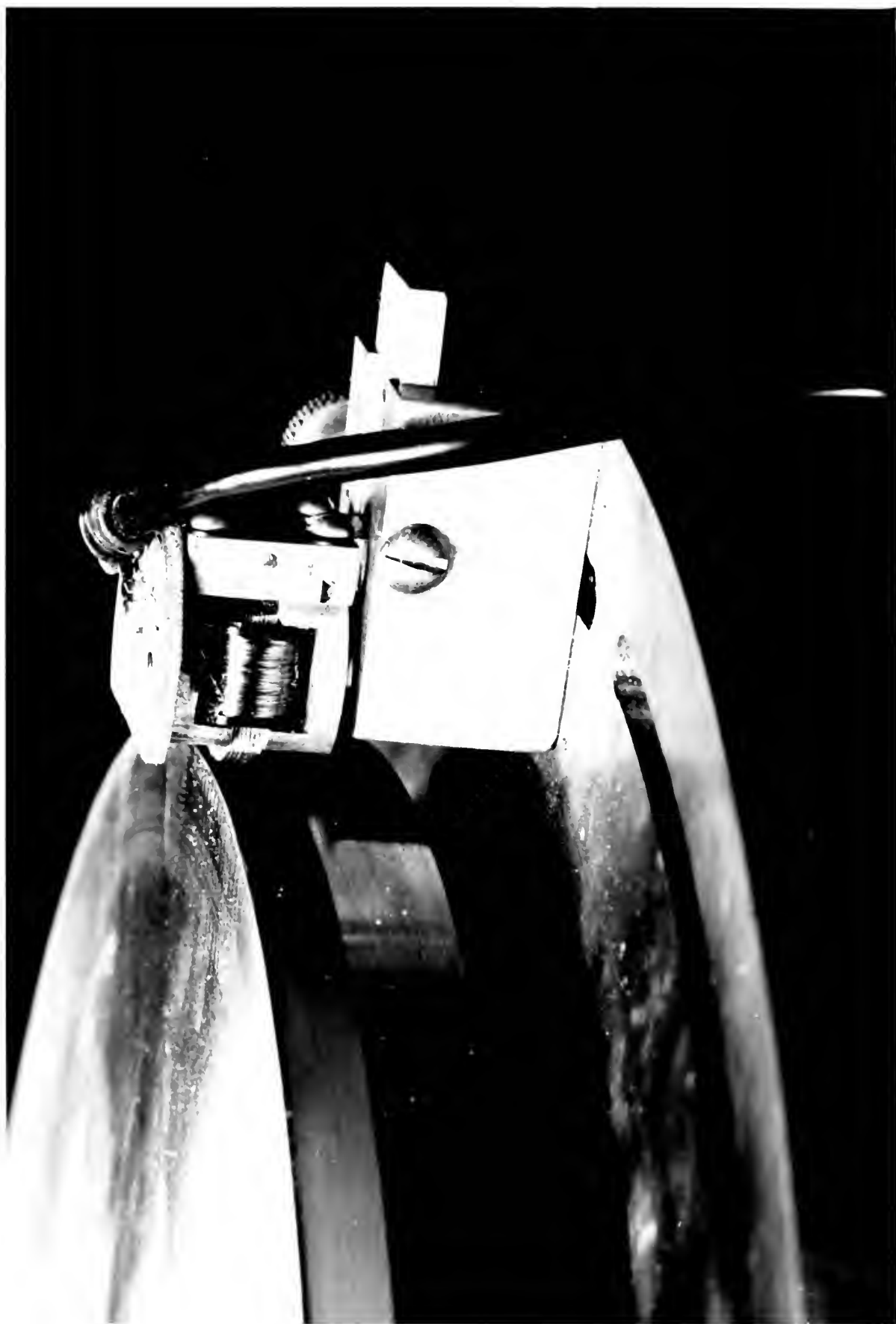
The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report.

The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report. The investigation was carried out by the Police and the Health and Safety Executive (HSE). The results of the investigation are set out in this report.











uneven surface of the tape. A number of various glues were used including normal household stationary glue, Ducco household cement, leather glue thinned with lacquer thinner, and rubber cement. The last glue was by far the best for this use, but even then an uneven surface was obtained. As measured by a dial indicator, irregularities of the order 0.0002 - 0.0004 were obtained. In addition, care had to be taken lest an imperfect bond result in blister like separations of the tape and the disc. These blisters would not show up on the dial indicator but would nevertheless be present, particularly when the disc revolved. The rubber cement was thinned slightly with lacquer thinner in order to obtain a more uniform spreading of the glue, but in such instances the blisters became excessive. A rather uniform thickness of the cement was obtained by revolving the disc while holding a brush wet with glue up against the periphery of the disc. By carefully working the tape from the centers towards the ends, squeezing out the excess glue, and by insuring that there was good adhesion over the entire circumference of the disc, an acceptable bond of the tape to the disc was obtained in the light of the amount of inherent run-out of the disc. The butt joint was placed at the low point of the disc.

Several attempts were made to position the reproduce heads so that it would just touch the tape at the high point of the disc. However, the head tended to tear off the tape when actual contact was made. In addition it was feared that this repetitive solid contact (the tape acting as a poor cushion between the head and disc) might permanently magnetize the reproduce head. Thus a deliberate gap was introduced between the tape on the disc and the reproduce head. This gap was less than 0.0005 inches as indicated by a feeler gauge.

The reproduce head used was a Brush Magnetic Recording Company BK - 919A with the following characteristics:

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

... ..

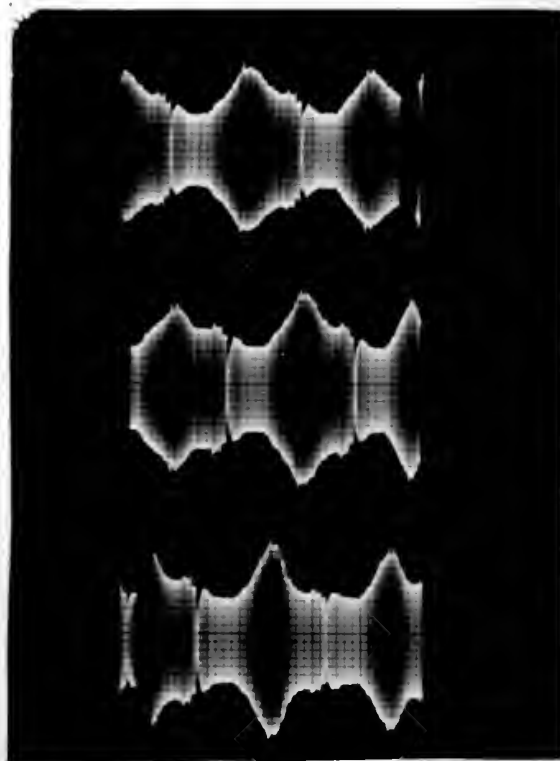
... ..

Pole width	.120 inches
Pole face gap	.0005 inches
Total coil resistance	120 ohms
Total impedance at 1800 cycle sec	1950 ohms
Total impedance at 40,000 cycle/sec	37,000 ohms
Maximum output level at 1000 cycle/sec and 7.5 inches/sec	0.004 volts with red ox

The effect of the eccentricity of the disc is shown in an oscilloscope photograph in Figure E.3. This picture is of the output of the tuned circuit for a frequency of 1930 cycles per second. The recorded frequency was 300 cycles per second. It can be seen that the variations in air gap between the reproduce head and the tape cause a variation of about 50% in this case. A more important consideration is that the variation between the high and low points on the disc will in itself vary with the frequency of the recorded component which is being analyzed. The effect of a higher resolution for the system ( a higher Q) can be also seen from these photographs. The higher Q filter accentuates the transient nature of this variation. This figure and Figure E.4 show the effect of the butt joint on the response of the filter. The butt joint in this case was about 0.008 inch. In several cases a butt joint was obtained which resulted in an almost imperceptible transient, and when the butt joint was being used to mark the revolutions of the disc, it was necessary to enlarge the width of the butt joint in one instance. The effect of the irregular surface of the tape can be seen from Figure E.5 which is an oscillograph of the recorded signal as amplified. The recorded frequency was 300 cycles/

1. The effect of the concentration of the gas is shown in an oscillograph photograph in Figure 2.3. This shows the effect of the concentration of the gas on the output of the gas cell. The concentration of the gas is varied from 0.1% to 1.0% and the output of the gas cell is measured in terms of the area under the curve. The area under the curve is proportional to the concentration of the gas. The output of the gas cell is measured in terms of the area under the curve. The area under the curve is proportional to the concentration of the gas.

The effect of the concentration of the gas is shown in an oscillograph photograph in Figure 2.3. This shows the effect of the concentration of the gas on the output of the gas cell. The concentration of the gas is varied from 0.1% to 1.0% and the output of the gas cell is measured in terms of the area under the curve. The area under the curve is proportional to the concentration of the gas. The output of the gas cell is measured in terms of the area under the curve. The area under the curve is proportional to the concentration of the gas.



Q - 28

Q - 51

Q - 118

FIGURE E.3  
Effect of eccentricity  
of disc on selected  
frequency

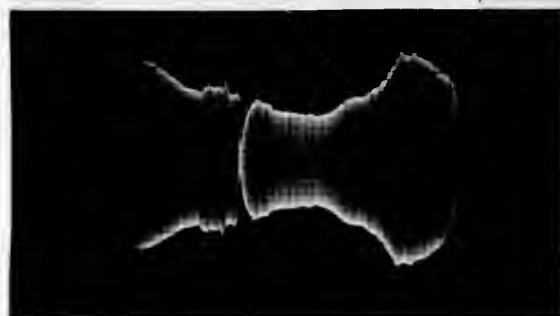
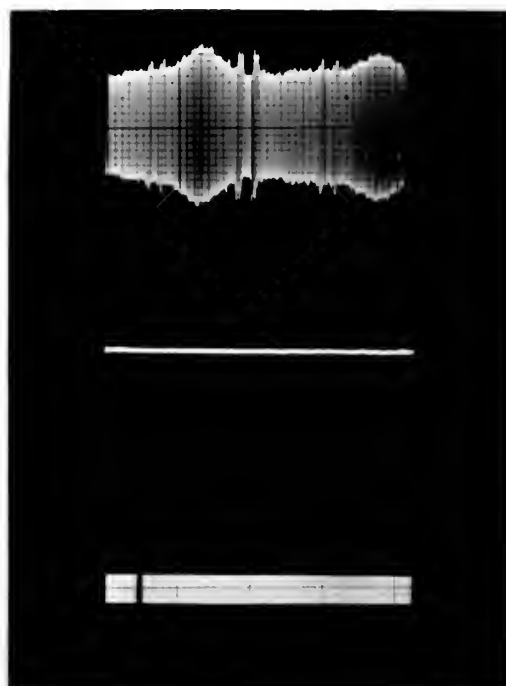


FIGURE E.4  
Butt joint





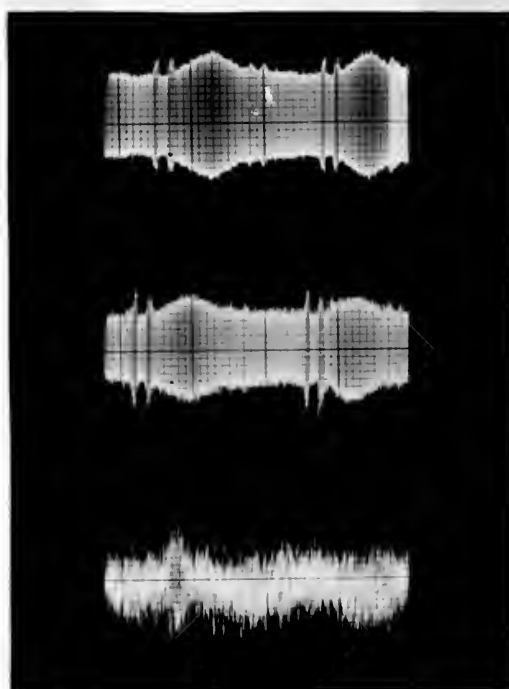


Signal as  
amplified

Amplifier  
noise

60 cycle  
reference

FIGURE E.5



Less than  
.001 in

Approximately  
.017 in

Approximately  
.039 in

FIGURE E.6  
Showing effect of  
air gap on signal  
noise,



second and the disc was being run at a speed which made this frequency look like 1950 cycles/second. The amplifier noise level and a 60 cycle/second reference are also shown. In this case the butt joint did not show up but the effect of two blisters did. These two transients were shown not to be related to the butt joint by noting in which part of the disc revolution these two transients occurred and then noting when the butt joint came up to the reproduce head. Two blisters were seen to coincide with the position of these transients. The tape used for this photograph was several days old and some of the rubber cement had dried without holding the tape to the disc. In many cases no blisters were present.

It was thought that the effect of the variation due to the eccentricity of the disc and the effect of irregularities of the tape surface might be lessened by setting the reproduce the head back from the tape. Thus the ratio of the air gap maximum to air gap minimum would be decreased. Figure E.6 shows the effect of increasing the air gap. Transients due to blisters have been accentuated rather than decreased. At a air gap of 0.017 inches the variation of the eccentricity of the disc has not been effectively smoothed. At a larger air gap setting the noise of system has overcome the signal, and it does not appear that the variation of the signal due to the eccentricity of the disc has been smoothed a great deal. In these three photographs of Figure E.6 the magnitude of the signal can not be compared since the gain of the amplifier and oscilloscope has been adjusted to give an indication which could be photographed. Actually the attenuation associated with a large air gap would require more gain and would result in a smaller signal to noise ratio.

Ward-Leonard System

The description and analysis of the Ward-Leonard system for speed

The description and analysis of the hand-drawn system for speed  
 and would result in a smaller signal to noise ratio.  
 the attention associated with a large air gap would require more gain  
 been adjusted to give an indication which could be photographed. Actually  
 not be compared since the gain of the amplifier and oscilloscope has  
 In these three photographs of Figure 5.6 the magnitude of the signal can  
 signal due to the eccentricity of the disc has been smoothed a great deal.  
 has overcome the signal, and it does not appear that the variation of the  
 effectively smoothed. At a larger air gap setting the noise of system  
 0.017 inches the variation of the eccentricity of the disc has not been  
 blisters have been associated rather than decreased. At a air gap of  
 Figure 5.6 shows the effect of increasing the air gap. Transients due to  
 the ratio of the air gap maximum to air gap minimum would be decreased.  
 be lessened by setting the reproduce the head back from the tape. Thus  
 of the disc and the effect of irregularities of the tape surface might  
 It was thought that the effect of the variation due to the eccentricity  
 the tape to the disc. In many cases no blisters were present.  
 several days old and some of the rubber cement had dried without holding  
 position of these transients. The tape used for this photograph was  
 up to the reproduce head. Two blisters were seen to coincide with the  
 sion these two transients occurred and then nothing when the butt joint came  
 to be retained by the butt joint by nothing in which part of the disc revolu-  
 up but had effect of the blisters. These two transients were shown not  
 toward reference line shown. It has come the butt joint did not show  
 (and a of cycle).

Hand-drawn System

The description and analysis of the hand-drawn system for speed

control are adequately covered in the literature.<sup>34,35</sup> Figure E.1 shows the arrangement by which the motor of the Ward-Leonard system drove the disc assembly. The motor used was an Oster, type E75, 115 volts, 3650 RPM, 1/20 horsepower, shunt wound. The generator of the system was the same. The amplifier used with the system is shown in schematic form in Figure E.7 and E.8.

It was originally planned to drive this system as a servomechanism with the error signal being derived by comparing the control signal to the speed of the motor as measured by a tachometer. Figure E.1 also shows a tachometer mounted with the motor. Several D.C. tachometers were used, but all operated improperly; particularly disadvantageous was their fluctuation in terminal voltage at low speeds. It was decided to run the Ward-Leonard system open loop using the speed control characteristics of the DC motor. Because of the small variation in load torque (except at low speeds when the coulomb friction contributes to a large percentage of the load torque) this plan was thought to be feasible. The transfer function between the controlling voltage at the servo amplifier input and the speed of the disc as measured by a speed counter is given in Figure E.9. The relationship between the controlling voltage and the output of the power amplifier (the generator field current) is also shown in Figure E.9. This figure indicates that a linear relationship holds for about a speed variation of four times (from 15 RPM to 60 RPM). For a complete analyzer such a limitation would not be desirable, but for this investigation we proceeded with this limited range of variation. As it happened this limited range was not a controlling factor in the investigation. When we compared several trial runs with the system, it was discovered that different results were being obtained with no change in the various parameters which should control the end

Figure 1.1 shows the block diagram of the system. The system is a closed-loop control system with a feedback path.

The system is a closed-loop control system with a feedback path. The system is a closed-loop control system with a feedback path. The system is a closed-loop control system with a feedback path.

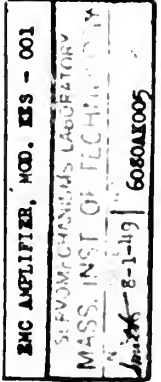
It was originally planned to drive this system as a servomechanism with the error signal being derived by comparing the control signal to the speed of the motor as measured by a tachometer. Figure 1.1 also shows a tachometer connected with the motor. Several D.C. tachometers were used, but all operated improperly, particularly at low speeds, and their function in terms of voltage at low speeds. It was decided to run the tachometer system open loop using the speed control character-

istic of the D.C. motor. Because of the small variation in load torque (except at low speeds when the control system contributed to a large percentage of the load torque) this plan was thought to be feasible.

The transfer function between the controlling voltage at the servo amplifier input and the speed of the motor as measured by a speed counter is given in Figure 1.2. The relationship between the controlling voltage and the output of the power amplifier (the generator field current) is also shown in Figure 1.2. The figure indicates that a linear relationship holds for about a speed variation of 10% from 10 RPM to 60 RPM. For a complete study, a much larger variation would be desirable, but for this investigation no provision was made for this range of variation. As it happened this limited range was not a controlling factor in the system. When we measured several other systems the response of the system varied from this and results were being obtained with an average of the system parameters as a control signal.

Figure 1.1 shows the block diagram of the system. The system is a closed-loop control system with a feedback path. The system is a closed-loop control system with a feedback path. The system is a closed-loop control system with a feedback path.

FIGURE E.7



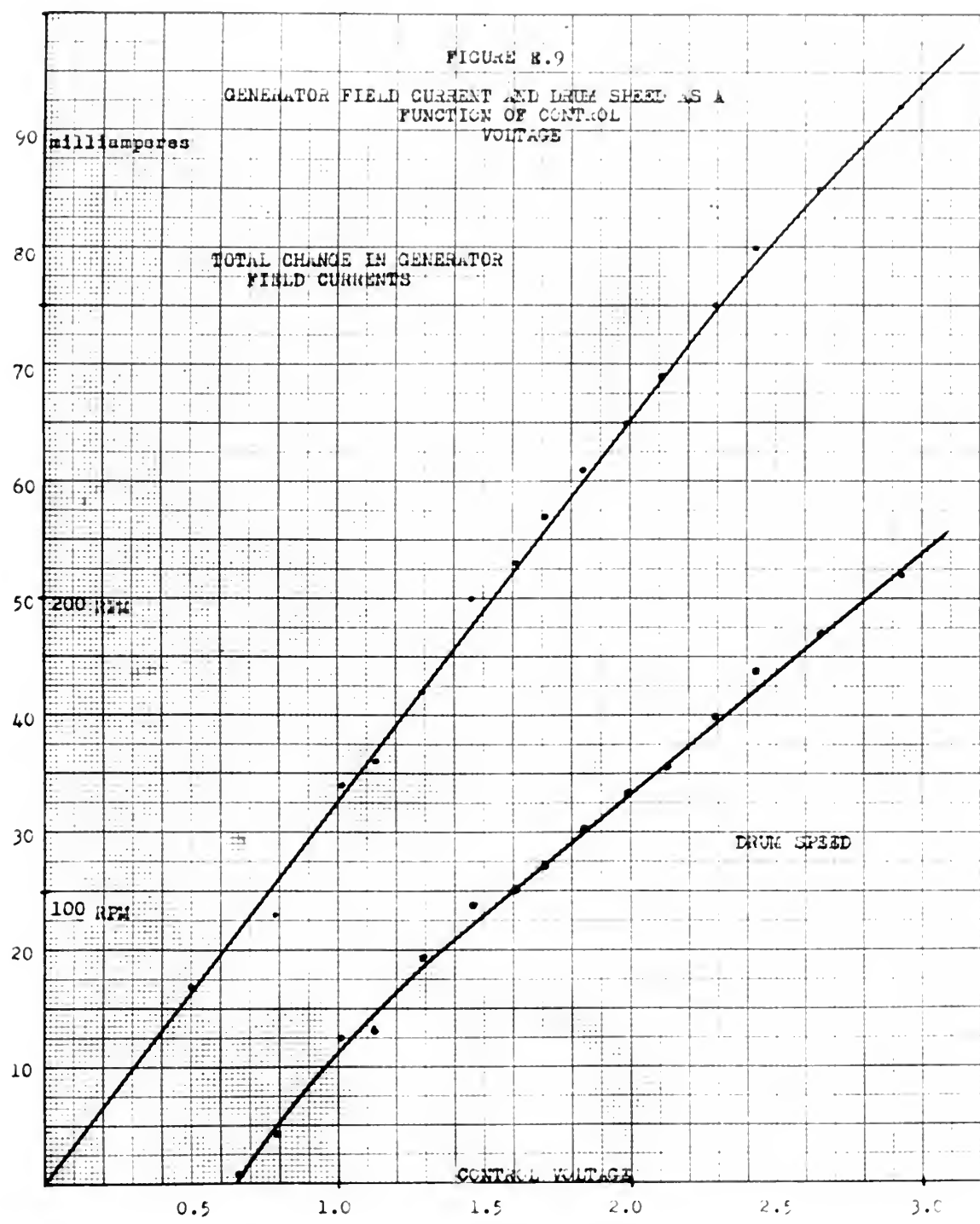
(continued on next page)













results. Figure E.11 shows the results of three different runs for a recorded frequency of 300 cycles per second,  $Q$  of 22. It was thought that the differences between the three indications were due to fluctuations in the speed of the disc as it was slowed down by the controlling voltage. Therefore a test was made with a constant controlling voltage. Figure E.10 shows the results of this test. Ideally a constant controlling voltage would result in a constant disc speed and a consequent steady output voltage. With the variation of the signal due to the eccentricity of the disc the output indication would necessarily be periodic. The results of this variation can easily be seen in Figure E.10. However, in addition to this periodic variation there is an erratic variation of the output which can only be explained as the erratic variation in input frequency to the tuned circuit. That this variation was due to the signal coming from the magnetic tape was shown by obtaining a steady response from the selective system and detector using a steady signal from an audio oscillator.

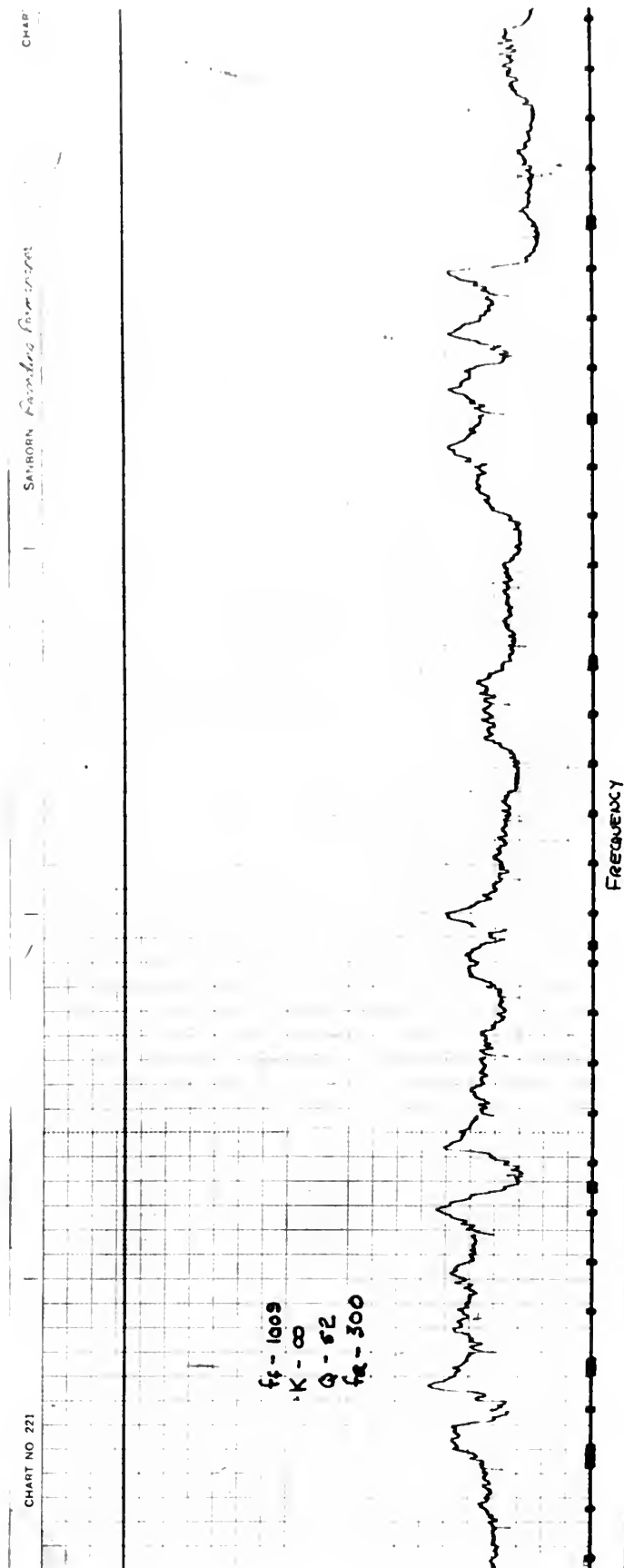
In order to obtain a smoother variation of the disc speed it was decided to disconnect the disc from the driving motor and having sped the disc up to speed, (using a cord) to let the disc slow down freely. Due to the large moment of inertia of the disc, the disc should act as a filter for small variations in torque such as that from the bearings on the disc shaft. A smoother variation was obtained as is indicated by the pulses of Figure 3.9. We no longer had control of the speed of the disc (a variable damping arrangement could control the free running speed somewhat), but control of the parameter  $K$  was possible by changing the  $Q$  of the tuned circuit. The speed-time relationship for the free

the all time low level of the frequency was for a  
frequency of 100 cycles per second, or 100 Hz. It was thought  
that the frequency between the two wheels was due to friction-  
torque in the drive of the disc as it was slowed down by the controlling  
voltage. Therefore a test was made with a constant controlling  
voltage. Figure 3.11 shows the results of this test. Ideally a constant controlling  
voltage would result in a constant disc speed and a consequent steady  
output voltage. With the variation of the signal due to the eccentricity  
of the disc the output indication would necessarily be periodic.  
The results of this variation can easily be seen in Figure 3.10. However,  
in addition to this periodic variation there is an erratic variation of the  
output which can only be explained as the erratic variation in input fre-  
quency to the tuned circuit. That this variation was due to the signal  
coming from the magnetic tape was shown by obtaining a steady response  
from the selective system and detector using a steady signal from an  
audio oscillator.

In order to obtain a smoother variation of the disc speed it was  
decided to disconnect the disc from the driving motor and having speed  
the disc up to speed, (using a cord) to let the disc slow down freely.  
Due to the large moment of inertia of the disc, the disc should not as  
a matter of fact vary in torque even as that from the bearings  
on the disc itself. A smoother variation was obtained as is indicated  
by the graph of Figure 3.2. No further control of the speed of  
the disc (a variable damping arrangement would control the free running  
speed somewhat), but control of the generator X was possible by changing  
the  $\phi$  of the tuned circuit. The speed-time relationship for the free

FIGURE E.10

Variation of filter output with  
constant controlling  
voltage







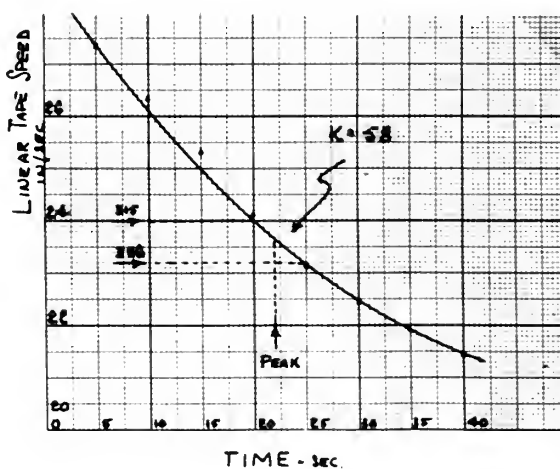
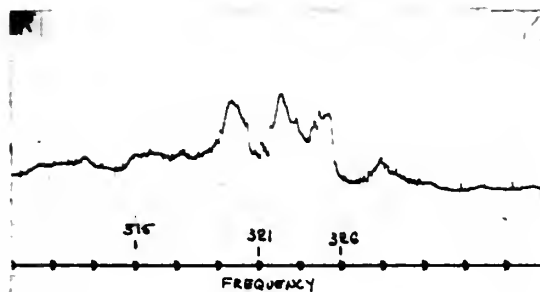
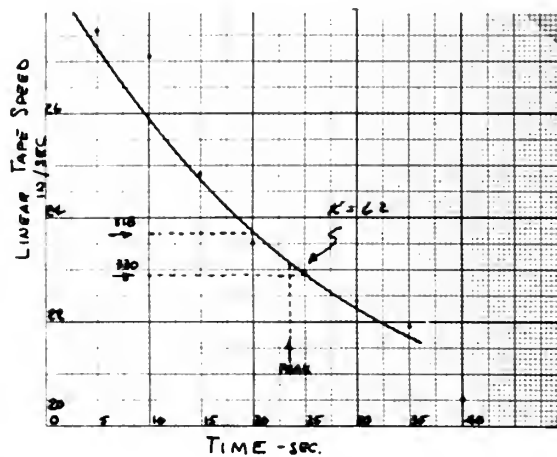
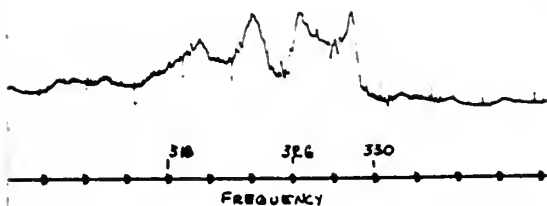
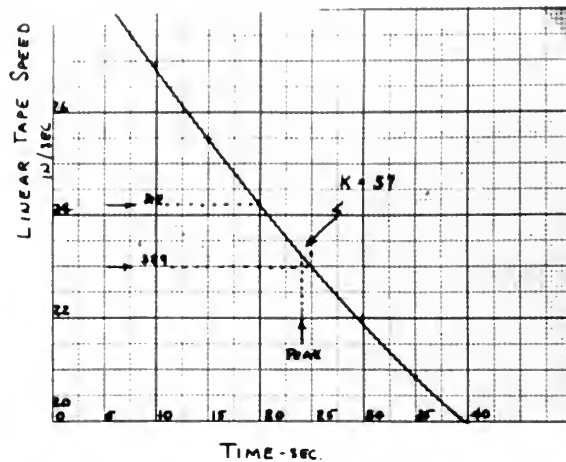
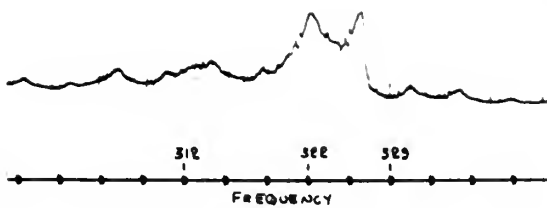


FIGURE 3.11

Variation of detected filter output for three different runs. All parameters remaining constant. Exponential frequency sweep.



running disc was determined by using the butt joint transient as a marker. Figure 3.9 also shows, in the middle recording, the marker recording at the top. The bottom trace shows one second marker pulses which were generated by the recorder (A Sanborn Cardette). These one second intervals were checked by counting the number over a measured period of time. The actual speed-time relationship was found by measuring the intervals between butt joint transients using a pair of dividers, and then calculating the speed of the disc knowing the circumference and the calibrated time scale of the recording. This was done for a number of points and a smooth curve was drawn through the points. The speed time relationships as determined in this manner are shown on Figures E.11, 3.8, and 3.9. The curve of figure E.11 were due to an exponential control voltage but were obtained using the procedure described above.

#### Selective System and Detector

A schematic of the selective System and detector is shown in Figure E.12. A parallel resonant LRC circuit with Q multiplication was used as the selective system. The Q of the resonant circuit is effectively multiplied by positive feed back from the vacuum tube which in effect cancels part of the resistance associated with the tank circuit. This circuit has good stability desired for a measurement circuit. The output of the Q multiplier is taken across the cathode resistor. A cathode follower isolation stage followed the Q multiplier. This stage was used to isolate the Q multiplier from the detector since any variations across the cathode resistor of the Q multiplier are reflected back to the tank circuit. The detector was a linear detector the design of which is covered in most elementary electronics textbooks. The decay time constant of this detector was determined by trial and error since the shape of the modulated signal

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

...the ... of the ...

### Defective System and Detector

A schematic of the defective system and detector is shown in Figure

E.12. A parallel resonant LC circuit with a multiplication was used as

the defective system. The Q of the resonant circuit is effectively infinite

and the defective load from the various coils which is affected by the

part of the resistance associated with the tank circuit. This circuit

has good stability for a measurement circuit. The output of the

Q multiplier is taken across the cathode resistor. A cathode follower

isolation stage followed the Q multiplier. This stage was used to isolate

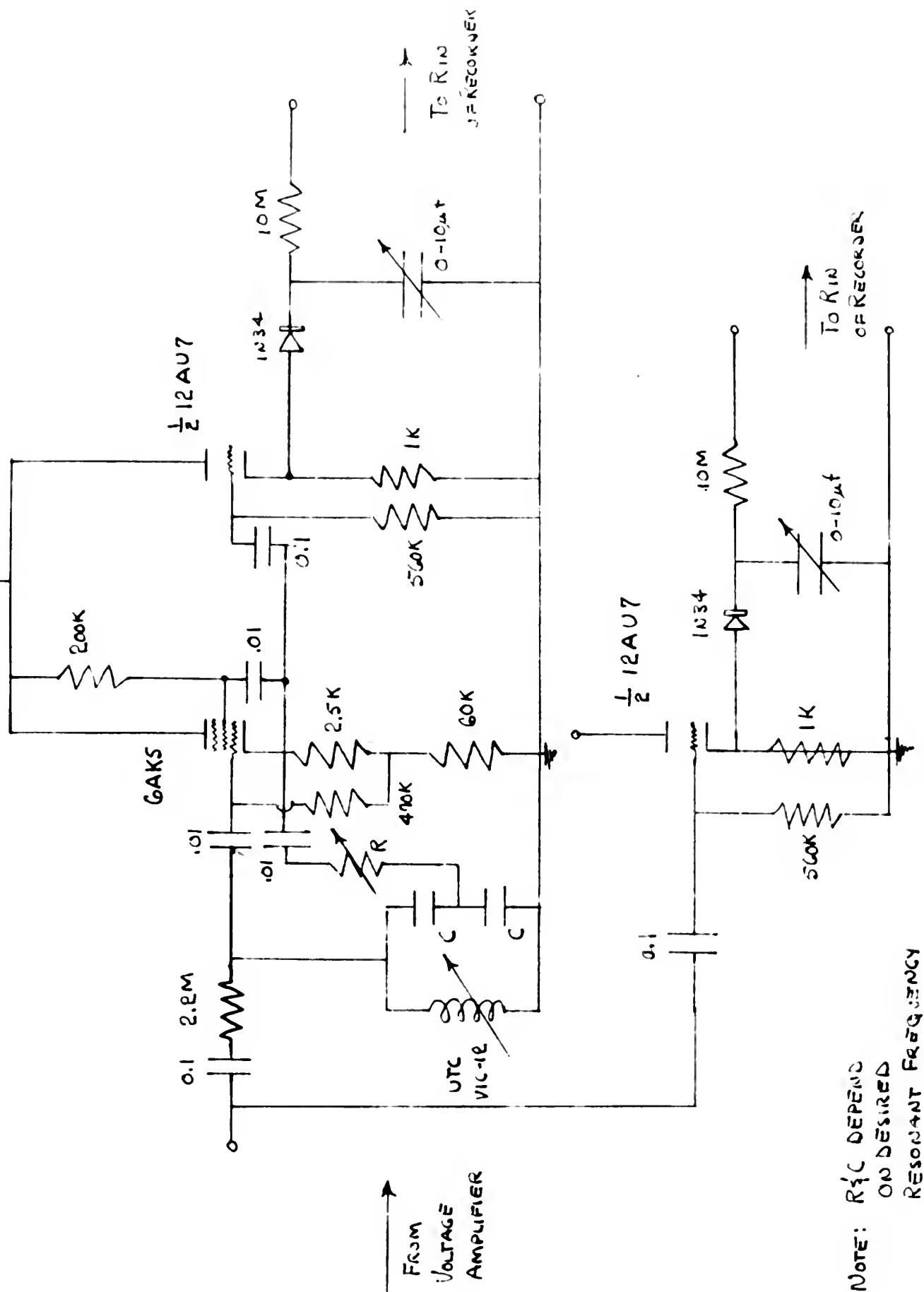
the Q multiplier from the detector since any variations across the cathode

resistor of the Q multiplier are reflected back to the tank circuit. The

detector was a linear detector the design of which is covered in most

elementary electronic textbooks. The heavy time constant of this detector

was compensated by a fast and slow time constant of the modulated signal





was not known with any exactitude. This time constant was set to show any fluctuations in the envelope of the curve due to the transient response of the filter.

The detector for the butt joint markers was of similar design. The input of this detector was taken off the amplified reproduced signal since it was desired to mark the recording for the entire sample length and not just for those time intervals when a reproduced frequency was equal to the frequency of the tuned circuit.

### Amplifier

The amplifier used was a General Radio type 714A voltage amplifier. The gain is adjustable between 20 to 80 decibels. The noise level of the amplifier in comparison to the amplified reproduced signal can be seen in Figure E.5. (It must be remembered that this signal is attenuated due to the air gap between the head and the tape in the order of 33 decibels from the 0.004 volts maximum possible. This amplifier has a flat frequency response between 20-18,000 cycles. The amplifier was very susceptible to vibrations when operated near its maximum voltage gain (A horn honking outside would produce a large output) The amplifier was removed from the table holding the drum assembly and the top was removed from the amplifier. The top was acting like a baffle and was being capacitively coupled to the grids of the tubes which were only a half inch or so below the top. (No more trouble was had with horns) The vibrations of the table carrying the Ward-Leonard generator and associated motor was excessive even in that they caused the reproduce head to vibrate noticeably. This assembly was moved to another table and both assemblies were put on rubber sheets which reduced the vibrations markedly.

The words "at" and "in" are used interchangeably in the text.

It is suggested that the word "in" be used throughout the text.

THESE WORDS ARE USED IN THE TEXT

Large new transparent hemispheres a week elevated with each set that you see

1992年12月

The amplifier used was a General Radio type 15A voltage amplifier. The gain is adjustable between 20 to 80 decibels. The noise level of the amplifier in comparison to the amplified reproduced signal can be seen in Figure 3.7. It must be remembered that this signal is attenuated due to the air gap between the head and the tape in the order of 33 decibels from the 0.005 volt maximum possible. This amplifier has a flat frequency response between 20-15,000 cycles. The amplifier was very susceptible to vibrations when operated near its maximum voltage gain. (A horn honking outside would produce a large output). The amplifier was removed from the table holding the drum assembly and the top was removed from the amplifier. The top was acting like a bell and was being capacitively coupled to the bridge of the table which was only a millimeter or so below the top. (No more trouble was had with hovers). The vibration of the table carrying the Ward-Johnson generator is substantially below the sensitive level in that they cannot be measured need to ultra sensitivity. This assembly was moved to another table and both assemblies were put on rubber bases which reduced the vibration sensibility.



### Frequency Measurements

A General Radio frequency meter, type 1141-A, was used as a master frequency standard. This meter is of the null detector type and requires use of head phones to determine the null. The manufacturer claims that the meter is accurate within 1%. No check was made of this frequency meter. Within the range of interest, 200 cycles to 5000 cycles, the null point could be detected to the accuracy of an imperceptible movement of the dial. However, the accuracy of the measurement was limited by the accuracy with which the dial could be read. Up to 500 cycles this accuracy was estimated to be about 1 cycle.

A General Radio direct reading frequency meter, type 834-B, was used for rapid measurements in order to center in on the measurement by the null type meter.

The Q of the tuned circuit was measured by using an audio oscillator, an oscilloscope, and the frequency meter. The bandwidth of the system was taken simply as the frequency difference between the half-power points. No response curve was taken for the tuned circuit but it was noted that the curve was unsymmetrical about the maximum point.

### Possible Equipment

(a) Disc and reproduce head. This assembly is plagued by the trouble experienced by most workers with drums of this sort - the error introduced by the necessity of having a varying air gap due to the eccentricity of the drum. At the same time the irregularities of the surface would cause trouble even if there were no eccentricity. There is the possibility of using a loop of tape which would be carried across the reproduce head similar to a conventional tape recorder. The fast rewind speeds of tape recorders show that tapes can be transported across the head at speeds several times greater

The first of these is the fact that the accuracy of the measurement is limited by the accuracy of the frequency meter. The accuracy of the frequency meter is limited by the accuracy of the dial. However, the range of interest, 200 cycles to 300 cycles, the mill points could be detected to the accuracy of an imperceptible movement of the dial. Within its accuracy which is 1%. No check was made of this frequency meter. Within the range of interest, 200 cycles to 300 cycles, the mill points could be detected to the accuracy of an imperceptible movement of the dial. However, the accuracy of the measurement was limited by the accuracy with which the dial could be read. If to 200 cycles this accuracy was estimated to be about 1 cycle.

A General Radio direct reading frequency meter, type 834-E, was used for rapid measurements in order to check in on the measurement by the mill type meter.

The 1 of the direct circuit was measured by using an audio oscillator, an oscilloscope, and the frequency meter. The frequency of the system was taken simply as the frequency difference between the half-power points. No response curve was taken for the direct circuit but it was noted that the curve was symmetrical about the maximum point.

### Possible Experiments

(a) The first experiment was the measurement of the frequency of the system. This was done by using the frequency meter. The accuracy of the frequency meter is limited by the accuracy of the dial. However, the range of interest, 200 cycles to 300 cycles, the mill points could be detected to the accuracy of an imperceptible movement of the dial. Within its accuracy which is 1%. No check was made of this frequency meter. Within the range of interest, 200 cycles to 300 cycles, the mill points could be detected to the accuracy of an imperceptible movement of the dial. However, the accuracy of the measurement was limited by the accuracy with which the dial could be read. If to 200 cycles this accuracy was estimated to be about 1 cycle.

than normal 7.5 inch per second speed. However, there is probably some limit to the range of speeds available from this method due to breaking of the tape. Other possibilities exist such as some mechanism which would keep the head in intimate contact with the tape on the disc or drum by means of spring loading. With the widespread use of magnetic drums and the fact that this problem is common to almost all users of such drums, we hesitate to make any comments other than that this field should be investigated for some simple, practical and workable solution which has already proved itself.

(b) Driving assembly. This investigation showed that there is a real need for an accurate speed controlling mechanism. At first one would immediately think of a feedback system; however the application in this case is one which does not require the advantage of the feedback servo-mechanism in maintaining speed control with widely varying torques. In this application it is important that the instantaneous speed of the disc not vary within limits, and these limits are set by the resolution of the system. Any feedback system would require that the error signal correct the speed before the recorded frequency being analyzed at that instant pass out of the pass band of the filter. Further investigation would be necessary to determine whether this feedback system would prove satisfactory. The possibility of the freely running drum as was used in this investigation shows promise in that the speed of the drum varies smoothly and that additional damping could be introduced to control the rate of slowing down. However, such a proposal would require a marker system coupled with some additional means of providing a direct reading indication of the analyzed frequency. The method as used here to determine the frequency of the sample is too laborious for a practical system.



(c) Function generator. A function generator which works off a function masked on the face of an oscilloscope was obtained but was not used. This function generator is in wide use today and has proved itself. For this investigation which was limited to exponential and free running variation of speed, the generator was not needed. For exponential or linearly varying voltages a simple RC decay or sweep circuit generator are recommended. The use of cams is possible, and these cams were investigated. However, precision cams followed by a translatory potentiometer would cost as much as a functionally wound potentiometer. For the function necessary for the equal sample analysis a function wound potentiometer and constant speed drive is recommended.

The first method is to use a constant speed drive as recommended. This is the most accurate method and gives the best results. The second method is to use a variable speed drive. This is less accurate than the first method but is still acceptable. The third method is to use a constant speed drive with a variable load. This is the least accurate method but is still acceptable. The fourth method is to use a constant speed drive with a variable load and a variable speed drive. This is the most accurate method but is also the most expensive. The fifth method is to use a constant speed drive with a variable load and a variable speed drive and a variable load. This is the most accurate method but is also the most expensive.

## APPENDIX F

BIBLIOGRAPHY

1. Barber, N. F. and Ursell, F., "A Frequency Analyzer Used in the Study of Ocean Waves." Nature. 158: 329 (1946).
2. Barber, N. F. and Ursell, F. "The Response of a Resonant System to a Gliding Tone." Philosophical Magazine. 39: 345 (1948).
3. Barber, N. F. "The Optimum Performance of a Wave Analyzer." Electronic Engineering. 21: 175 (1949).
4. Beranek, L.L. Acoustic Measurements. New York: John Wiley and Sons, Inc., 1949.
5. Clavier, A.G. "Application of Fourier Transforms to Variable Frequency Circuit Analysis." Proceedings of the Institute of Radio Engineers. 37: 1287 (1949).
6. Ekstein, H. and Schiffman, T. "Response of a Linear Network to an Input with Linearly Variable Frequency as Obtained in Sweep Frequency Testing." Proceedings of the National Electronic Conference. 7: 454 (1951).
7. Gardner, M. F. and Barnes, J. L. Transients in Linear Systems. New York: John Wiley and Sons, Inc., 1942.
8. Guillemin, E. A. Communication Networks. Volume I. New York: John Wiley and Sons, 1931.
9. Hamilton, W. H. "The Response of a Tuned Amplifier to a Signal Varying Linearly in Frequency." Proceedings of the National Electronic Conference. 4: 377 (1948).
10. Hok, G. "Response of Linear Resonant Systems to Excitation of a Frequency Varying Linearly with Time." Journal of Applied Physics. 19: 242 (1948); Erratum: 19:623 (1948)
11. Horton, J. W. Ambiguity in Wave Analyzer Measurements. U.S. Navy Underwater Sound Laboratory Technical Memorandum Serial 905-27 (1951).
12. Horton, J. W. Laboratory Notebook. U. S. Navy Underwater Sound Laboratory. (1952)
13. Jastram, P. S. and McCouch, G.P. "A Video-Frequency Noise Spectrum Analyzer." Proceedings of the Institute of Radio Engineers. 37: 1127 (1949)
14. Lewis, F. M. "Vibration During Acceleration Through a Critical Speed." Transactions of the American Society of Mechanical Engineers. 54: 253 (1932).

# REFERENCES

1. Barker, H. S. and Israel, T. "A Frequency Analyzer Used in the Study of Ocean Waves." Journal of the Acoustical Society of America, 19: 2 (1947).
2. Barker, H. S. and Israel, T. "The Response of a Resonant System to a Changing Tone." Philosophical Magazine, 39: 392 (1948).
3. Barker, H. S. "The Optimum Performance of a Wave Analyzer." Measurement Engineering, 32: 172 (1945).
4. Bennett, L. D. Acoustic Measurements. New York: John Wiley and Sons, Inc., 1948.
5. Clavier, A. G. "Application of Fourier Transforms to Variable Frequency Circuit Analysis." Proceedings of the Institute of Radio Engineers, 37: 1287 (1949).
6. Eisele, H. and Schiffman, T. "Response of a Linear Network to an Input with Linearly Variable Frequency as Obtained in Sweep Frequency Testing." Proceedings of the National Electronic Conference, 7: 454 (1951).
7. Garney, H. F. and Bortner, J. L. Transforms in Linear Systems. New York: John Wiley and Sons, Inc., 1949.
8. Gullikens, E. A. Communication Networks. Volume I. New York: John Wiley and Sons, 1951.
9. Hamilton, W. H. "The Response of a Band Amplifier to a Signal Varying Linearly in Frequency." Proceedings of the National Electronic Conference, 4: 37 (1948).
10. Holt, G. "Response of Linear Network Systems to Excitation of a Frequency Varying Linearly with Time." Journal of Applied Physics, 19: 242 (1948); Abstract: 19:623 (1948).
11. Horton, J. W. "Reply to Wave Analyser Comments." U.S. Navy Hydrographic School Laboratory Technical Report 907-2 (1951).
12. Horton, J. W. Hydrographic Laboratory. U.S. Navy Hydrographic School Laboratory, 1952.
13. Jastrow, P. C. and Johnson, J. E. "A Wide-Bandwidth Noise Spectrum Analyzer." Proceedings of the Institute of Radio Engineers, 37: 1287 (1949).
14. Lewis, W. H. "Variable-Frequency Amplifier Through a Critical Speed." Proceedings of the Institute of Radio Engineers, 37: 1287 (1949).



15. Marique, J. "The Response of RLC Resonant Circuits to EMF of Sawtooth Varying Frequency." Proceedings of the Institute of Radio Engineers. 40: 945 (1952).
16. Meyer, E. "A Method for Very Rapid Analysis of Sound." Journal of the Acoustical Society of America. 7: 83 (1935).
17. Penfield, H. Development and Analysis of a Log  $f = kt$  Audio Oscillator. M.S. Thesis, M.I.T. (1952).
18. Sacia, C. F. "Photomechanical Wave Analyzer Applied to Inharmonic Analysis." Journal Optical Society of America. 9: 487 (1924).
19. Soanes, S. V. "Some Problems in Audio Frequency Spectrum Analysis." Electronic Engineering. 24: 263, 24: 312 (1952).
20. Soanes, S. V. "Comments on the Response of RLC Resonant Circuits by J. Marique." Proceedings of the Institute of Radio Engineers. 41: 935 (1953).
21. Williams, E. M. "Radio-Frequency Spectrum Analyzers." Proceedings of the Institute of Radio Engineers. 34: 18 (1946).
22. Cunnas, M. "Magnetic Recording Tapes". American Institute of Electrical Engineers' Technical Paper 48-75 (1947).
23. Gibbs, H. E. "Problems Involved in Magnetic Tape Recording." Audio Engineering. 38: 19 (1954).
24. Hastings, A. E. "Methods of Obtaining Amplitude-Frequency Spectra." The Review of Scientific Instruments. 23: 344 (1952).
25. Koenig, W., Dunn, H., and Lacy, L. Y. "The Sound Spectrograph." Journal of the Acoustical Society of America. 18: 19 (1946).
26. Latham, W. S. "A Study of Limitations of Magnetic Tape." U.S. Navy Underwater Sound Laboratory Report No. 140. (1951).
27. Lennert, F. "Equalization of Magnetic Tape Recorders for Audio and Instrumentation Applications." Transactions of I.R.E. Professional Group in Audio. AU-1: (1953).
28. Peterson, G. E. and Raisbeck, G. "The Measurement of Noise with the Sound Spectrograph." Journal of the Acoustical Society of America. 25: 1157 (1953).
29. "Frequency Analyzer Design" Electronic Equipment. 2: 2 (1954).
30. Terman, F. E. Radio Engineers' Handbook, 1st Edition. New York: McGraw-Hill Book Company, 1943.
31. Wallace, R. L. "The Reproduction of Magnetically Recorded Signals." The Bell System Technical Journal. 30: 1145 (1951).

- [illegible]

32. Harris, H. E. "Simplified Q Multiplier." *Electronics*. 24: 130 (May 1951).
33. Brown, G. S. and Campbell, D. P. Principles of Servomechanisms. New York: John Wiley and Sons, 1948.
34. Chestnut, H. and Mayer, R. W. Servomechanisms and Regulating System Design, New York: John Wiley and Sons, 1951.
35. McNee, A. B. "An Electronic Differential Analyzer." Research Laboratory of Electronics Technical Report No. 90. 12 (December 1948).
36. Sunstein, D. E. "The Photoformer", Paper Presented at National Convention of the I.R.E. (March 1948).

1. The first part of the report is devoted to a general survey of the situation in the field of atomic energy.

2. The second part of the report is devoted to a detailed analysis of the work of the International Atomic Energy Agency (IAEA) during the year 1954.

3. The third part of the report is devoted to a detailed analysis of the work of the IAEA during the year 1954, with special reference to the work of the IAEA in the field of atomic energy.

4. The fourth part of the report is devoted to a detailed analysis of the work of the IAEA during the year 1954, with special reference to the work of the IAEA in the field of atomic energy.

5. The fifth part of the report is devoted to a detailed analysis of the work of the IAEA during the year 1954, with special reference to the work of the IAEA in the field of atomic energy.











HL4 Haley

28832

A wave analyzer employing variable speed magnetic tape.

HL4 Haley

28832

A wave analyzer employing variable speed magnetic tape.

thesis #14

A wave analyzer employing variable speed



3 2768 002 13680 6

DUDLEY KNOX LIBRARY